Second North American Summer School in Language, Logic and Information Student Session Proceedings

Indiana University, June 17th-21st 2003
Student Session Chair: John Hale

Preface

One of the most wonderful parts of the North American Summer School for Language, Logic and Information is the opportunity it affords young scholars to discuss their work with experts. By the same token, it is often inspiring for established researchers to see the creative directions in which recent work is being taken.

From this vantage point, it is certainly a privilege to present to the NASSLLI community such an promising collection of student work. Small in number but high in quality, the 2003 Student Session is sure to be an exciting event. This excitement is tempered by the news of immigration difficulties that prevent one of the authors from attending the Session. This paper is nonetheless included in these proceedings.

Thanks are due to the many people who assisted with preparations for the Student Session. Among them are the redoubtable **Student Session Program Committee**

Julia Hockenmaier, University of Edinburgh
Gerhard Jaeger, Potsdam University
Greg Kobele, UCLA
Kai-Uwe Kühnberger, University of Osnabrueck
Jens Michaelis, Potsdam University
Yevgeniy Makarov, Indiana University
Gideon Mann, Johns Hopkins University
Ken Shan, Harvard University
Christian Skalka, University of Vermont
Rachel Sussman, University of Rochester

as well as the formidable **Local Arrangement Co-Chair**, Ignacio Viglizzo and our **Webmaster** Lei Qian.

Finally, our deepest thanks to Professor Lawrence S. Moss (Indiana) for his guidance.

John Hale
Baltimore, Maryland

Table of Contents

Yet Another Statistical Case Assignment in Korean

Kihwang Lee
School of Informatics, University of Edinburgh
k.lee@ed.ac.uk

Abstract

We present a simple statistical method of case assignment in Korean. We use a set of incomplete but abundant examples as our training material, which is extracted from a large unannotated corpus using a very simple procedure. Evaluation against human-annotated test data reveals that our simple approach can survive even if the training material is not perfect.

1 Introduction

Head-dependent relationships have been rigorously used in many large-scale language engineering tasks. For instance, studies on statistical treebank parsing (Collins 1999; Hockenmaier 2003) were able to achieve high levels of accuracy applying lexicalised probabilistic models over head-dependent tuples. Head-dependent tuples are also useful in applications including information extraction (Palmer et al. 1993; Yeh 2000) and Web document retrieval (Grefenstette 1997). Lafferty et al. (1992), Buchholz et al. (1999), and Carroll and Briscoe (2002) are some of the research efforts that applied various techniques to robust extraction of grammatical relations (SUBJECT, OBJECT, etc), a particular type of head-dependent relationships.

At first sight, acquiring head-dependent tuples in languages with explicit case marking systems including Korean looks quite trivial. It is, however, not always possible to get accurate head-dependent tuples, for a large number of nouns are used without case markers in Korean.[1] Syntactic structure by itself does not provide efficient information for head-dependent tuples since the position of an argument in a sentence is not fixed. Therefore, assigning proper cases to nouns without any case markers is a vital process for the head-dependent relation acquisition in Korean.

The goal of this study is to construct a simple statistical model of assigning cases for nouns used without any case particles. We use a set of incomplete but abundant training material prepared without a parser or any other high level language processors except a part-of-speech tagger. In particular, we are interested in combining the information from neighbouring words of the noun and the predicate in question. Similar but not identical approaches have been taken by several studies. We, however, consider that there still is a room for further investigation.

This paper is organised as following: Section 1 introduces the background and the goal of the paper. Examples of case marking and case particle omission in Korean are presented in Section 2. Section 3 briefly surveys previous work that are related to current study. In Section 4, our simple statistical model is described. Section 5 shows experimental settings including training material collection and evaluation method. The result of our

[1] Around 30% of nouns are used without case particles in the 38-million word Yonsei Corpus of Korean.

experiment and a discussion are presented in Section 6. Finally, Section 7 concludes this paper and highlights some of the issues that can be tackled in future work.

2 Case Marking in Korean

There are diverse theories on case assignment mechanism in Korean.[2] It is, however, widely accepted that the cases are realised by case markers or case particles in a similar way which also can be observed in other languages like Japanese and Turkish. Consider the following examples.

(1) a. Seho-ka cangnankam-ul pakwuni-ey neh-ess-ta.
Seho-NOM toy-ACC basket-LOC put-PAST-DECL
'Seho put a toy into a basket.'

b. Cangnankam-un Seho-to pakwuni-ey-man neh-ess-ta.
Toy-TOP Seho-also basket-LOC-only put-PAST-DECL.
'(lit.) As for toys, Seho also put them only in a basket.'

c. Seho-ka cangnankam-ul neh-un pakwuni.
Seho-NOM toy-ACC put-REL basket.
'The basket which Seho put a toy in.'

In (1a), case particles *ka, (l)ul*, and *ey* are marking NOMINATIVE, ACCUSATIVE and LOCATIVE cases respectively. On the other hand, case particles *ka, (l)ul* are omitted in (1b). In (1b), auxiliary noun particles *nun* and *to* are blocking explicit case markings by case particles.[3] In some cases, nouns can be used without any particles. Thus, when the case particles are not present, it is a matter of conjecture which case is involved. The position of a noun itself cannot give much help since Korean is a relatively free word order language.

Case particles for head nouns are also omitted in relative clauses as shown in (1c). The relativisor in Korean is nothing but an adnominal ending of a predicate and it does not bear any case information of the noun modified by the relative clause.

Case particles can be divided into three groups according to their behaviour regarding the case particle omission. First group of case particles are *ka* NOMINATIVE and *lul* ACCUSATIVE. These particles can never be used with auxiliary particles *nun* TOP and *to* 'also'. They are also frequently omitted, especially in colloquial speech. *Ey* LOCATIVE$_1$, *eyse* LOCATIVE$_2$, *ekey* DATIVE, and *lo* INSTRUMENTAL that belong to the second group of case particles. These particles can be used with or without auxiliary particles. They can also be omitted under certain conditions. Other case particles are always realised in surface and never omitted.

Naturally we are only interested in case particles belong to the first and the second groups. We merge *ey* LOCATIVE$_1$ and *ekey* DATIVE because *ey* can also be used as DATIVE marker and it is not easy to distinguish between the two different usages. The only difference between *ey* and *ekey* is that the former is used with inanimate nouns while the latter is used with animate nouns (Lee and Ramsey 2000). As a result, we have five target case particles for the task of case assignment: *ka* NOMINATIVE, *lul* ACCUSATIVE, *ey* LOCATIVE$_1$, *eyse* LOCATIVE$_2$ and *lo* INSTRUMENTAL.[4]

[2]For a general introduction to Korean, refer to Sohn (1999) and Lee and Ramsey (2000).
[3]Auxiliary particles are used to express semantic/pragmatic content.
[4]These, of course, are baseforms of bunch of phonetic/stylistic variants.

3 Previous Work

Yang and Kim (1993, 1994) are the first studies that adopted a statistical method to resolve case ambiguity in Korean. In these studies, case decision was guided by SR (Statistical Relevance) score, which is the sum of SS (Subcategorisation Score) and CS (Co-occurrence Score). These scores are calculated from the frequency counts of p (predicate), n (noun) and j (particle or *josa*) as shown in the following equations.

$$SR = SS + c * CS, \quad c > 1$$
$$SS(p) = \frac{f(p,j)}{f(p)}, \quad j \in \{\text{NOMINATIVE}, \text{ACCUSATIVE}\}$$
$$CS(n,p) = \frac{f(n,p,j)}{f(n,j) + f(p,j) - f(n,v)}$$

Kim (1996) introduced *Association measure* influenced by Resnik (1993). This measure was defined as multiplication of the conditional probability of a noun given a predicate and a particle, and the conditional mutual information of the predicate and the noun given the particle as following:

$$Assoc(p,j,n) = P(n|p,j)I(p;n|j)$$
$$I(p;n|j) = \log_2 \frac{P(p,n|j)}{P(p|j)P(n|j)}$$

Training examples were semi-automatically collected from a corpus. This work used a class-based smoothing technique to deal with unseen $\langle p, n \rangle$ pairs using an experimental thesaurus (Lim 1993).

Li et al. (1998) extracted triplets of $\langle p, n, j \rangle$ and generalised them into conceptual representations using a bilingual dictionary and a Japanese thesaurus (Ohno and Hamanishi 1981). Case decision is done by choosing a case that maximises the conceptual similarity of the predicate and the noun.

Chung (1999) also used an *Association measure*. In this work, training examples are collected in a full automatic method from a corpus. This study clustered nouns using another experimental thesaurus Cho and Ok (1997) for a class-based smoothing. *Association measure* in this study is defined as following:

$$Assoc(p,n,j) = \alpha \times \overline{Assoc}(p,n,j) + (1-\alpha) \times \overline{Assoc}(p.j) \qquad (0.5 \leq \alpha \leq 1)$$
$$\overline{Assoc}(p,n,j) = \max\left(P(n,j|p), \frac{P(class(n),j|p)}{N}\right)$$
$$\overline{Assoc}(p,j) = P(j|p)$$

In summary, all previous work collected triplets of $\langle p, n, j \rangle$ and used relatively simple methods, whereby case decision is basically dependent on the predicate and the noun. The scoring functions seems to be quite ad hoc. Some of the studies utilised class-based smoothing techniques to overcome the data sparseness problem.

4 Statistical Models for Case Assignment in Korean

We regard the statistical case assignment problem as a classification task. We build a statistical classifier which maps a tuple of feature values into a case particle which maximises the score assigned by a particular statistical model. This classifier is formulated in Equation (1) together with our choice of a feature set for the classification task.

$$cl_M(n,p,d,s) = \text{argmax}_{j\in\mathcal{J}} M(j,p,n,d,s) \quad (1)$$

where

- M: statistical model
- $\mathcal{J} = \{ka \text{ NOM}, lul \text{ ACC}, ey \text{ LOC}_1, eyse \text{ LOC}_2, lo \text{ INST}\}, j \in \mathcal{J}$
- p: predicate
- n: noun (general noun, bound noun, pronoun, numeric, etc.)
- d: distance between n and p (far, middle, near)
- s: pseudo-subcategorisation pattern (concatenation of neighbouring case particles)

The four features p, n, d, s can be easily obtained from a set of sentences. We don't use other neighbouring words such as other nouns at the moment. Our belief is that a good combination of these features will give us a firm ground that further steps can be built on.[5]

Now we describe our statistical model and Naive Bayes classifier, a well-known simple classifier.

4.1 Simple Probabilistic Model with a Back-Off Smoothing

One simple way of combining individual features is describing a training instance as a joint event of the features and the decision. This model cl_{BO} chooses the most probable case particle which maximises the joint probability of $\langle j,p,n,d,s\rangle$ as shown in Equation 2.

$$cl_{BO}(p,n,d,s) = \text{argmax}_{j\in\mathcal{J}} P(j,p,n,d,s) \quad (2)$$

We can factor $P(j,p,n,d,s)$ as (3). From this equation, we can simplify the classifier cl_{BO} as (4) since we are not interested in the actual value of the joint probability.

$$P(j,p,n,d,s) = P(s)P(d|s)P(n|d,s)P(p|n,d,s)P(j|n,p,d,s) \quad (3)$$
$$cl_{BO}(p,n,d,s) = \text{argmax}_{j\in\mathcal{J}} P(j|n,p,d,s) \quad (4)$$

To estimate $P(j|n,p,d,s)$, we use frequency counts of tuples $\langle j,p,n,d,s\rangle$ and $\langle p,n,d,s\rangle$. We adopt a simple back-off (Katz 1987) style smoothing technique which was effectively applied to prepositional phrase attachment in English (Collins and Brooks 1995) and grammatical relation assignment in German (de Lima 1997). We back-off to a combination of tri-grams when we don't have any counts for 4-grams, and to a combination of bi-grams and so on as shown in Equation (5).

[5] Examples of extracted features are shown in Table 1.

$$P(j|n,p,d,s) = \begin{cases} \frac{f(j,n,p,d,s)}{f(n,p,d,s)} & \text{if } f(n,p,d,s) > k \\ \frac{f(j,p,n,d)+f(j,p,n,s)+f(j,p,d,s)+f(j,n,d,s)}{f(p,n,d)+f(p,n,s)+f(p,d,s)+f(n,d,s)} & \text{if } f(p,n,d)+f(p,n,s)+f(p,d,s)+f(n,d,s) > k \\ \frac{f(j,p,n)+f(j,p,d)+f(j,p,s)+f(j,n,d)+f(j,n,s)+f(j,d,s)}{f(p,n)+f(p,d)+f(p,s)+f(n,d)+f(n,s)+f(d,s)} & \text{if } f(p,n)+f(p,d)+f(p,s)+f(n,d)+f(n,s)+f(d,s)>k \\ \frac{f(j,p)+f(j,n)+f(j,d)+f(j,s)}{f(p)+f(n)+f(d)+f(s)} & \text{if } f(p)+f(n)+f(d)+f(s) > k \\ \frac{f(p)+f(n)+f(d)+f(s)}{N+N+N+N} & \text{otherwise} \end{cases} \quad (5)$$

We have experimented with other combination methods. However, the above method which was proposed by Collins and Brooks (1995) also worked best for us.

4.2 Naive Bayes Classifier

The Naive Bayes classifier is a simple stochastic classification method originally described in Duda and Hart (1973). The Bayesian approach to classifying a a new instance is to assign the most probable target value v_{MAP}, given the conjunction of the feature values $a_1, a_2, \ldots, a_n$ that describe the instance.[6]

$$v_{MAP} = \text{argmax}_{v_j \in V} P(v_j|a_1, a_2 \ldots a_n) \quad (6)$$

$$= \text{argmax}_{v_j \in V} \frac{P(a_1, a_2 \ldots a_n|v_j)P(v_j)}{P(a_1, a_2 \ldots a_n)} \quad (7)$$

$$= \text{argmax}_{v_j \in V} P(a_1, a_2 \ldots a_n|v_j)P(v_j) \quad (8)$$

We could attempt to estimate the two terms in Equation (8) based on frequency counts from the training data. Estimating the first term $P(a_1, a2, \ldots, a_n|v_j)$, however, is not not feasible unless we have a very large set of training data which can give us enough counts for the all the different $P(a_1, a2, \ldots, a_n|v_j)$ terms. The number of these terms is equal to the number of possible instances times the number of possible target values. To handle this problem, we make a simple assumption that the feature values are conditionally independent given the target value. In other words, we simplify the probability of observing the conjunction of feature values $(a_1, a_2, \ldots, a_n)$ as the product of the probabilities for the individual attributes as shown in Equation (9). As a result, we have the Naive Bayes classifier in Equation (10) by substituting the first term in Equation (8).

$$P(a1, a2, \ldots, a_n|v_j) = \prod_i P(a_i|v_j) \quad (9)$$

$$v_{NB} = P(v_j)\, \text{argmax}_{v_j \in V} \prod_i P(a_i|v_j) \quad (10)$$

If we apply this classifier to our task, we have following Naive Bayes classifier for the statistical case assignment task in Korean.

$$cl_{NB}(n, p, d, s) = \text{argmax}_{j \in \mathcal{J}} P(j)P(n|j)P(p|j)P(d|j)P(s|j) \quad (11)$$

[6]We follow the description of the Naive Bayes classifier in Mitchell (1997).

5 Experimental Settings

We present our training data construction and evaluation methods in this section.

5.1 Training Data Preparation

As stated in section 4, our training data should be a set of case assignment examples in the form of tuple $\langle j, n, p, d, s \rangle$. One natural way of acquiring such data is to use annotated corpora. However, case annotated corpus of Korean is not available for the moment. For the given task, we can collect training examples from an unannotated corpus because a large amount of nouns occur with case particles. It is also required that the nouns do not have attachment ambiguities. Following the example of Ratnaparkhi (1998)'s unsupervised PP attachment, we use a very simple and cheap procedure for isolating sentence fragments from complex sentences avoiding the necessity of complex sentence fragmentation or full parsing. These sentence fragments can be regarded as rough replacements for simple sentences. The underlying assumption is that heuristically valid data collected from a sufficiently large unannotated corpus can serve as an adequate substitute for the knowledge that could have been obtained from a fully annotated corpus.

To isolate a sentence fragment, we start from the end of a sentence and move backwards to the start of the sentence recording every noun and target case particle pair if we can find any. Figure 1 is an outline of our procedure for the sentence fragment collection.

```
n ← the number of the wordforms in the sentence
wordfoms[] ← the array of the wordforms in the sentence
pred ← ''
noun_particles ← []
𝒥 = {NOM, ACC, LOC1, LOC2, INST}
if wordforms[n − 1] = predicate then
   pred ← stem of wordforms[n]
else
   Stop.
end if
n ← n − 1
while wordform[n] ≠ predicate do
   if wordform[n] = noun with a particle and particle ∈ 𝒥 then
      Add the noun and the particle pair to noun_particles.
   end if
   n ← n − 1
end while
if noun_pred ≠ [] then
   Output pred and noun_particles.
end if
```

Figure 1: Sentence fragment collection procedure

Using the above procedure, we extracted 1.36 million sentence fragments from the 38-million word Yonsei Corpora (Seo 1999). These corpora were tagged using a publicly available part-of-speech tagger (Kim 2001)[7] and sentence-divided by a piece of simple software. From the sentence fragments, we constructed 1 million training examples satisfying our choice of feature set. Table 1 shows our training data preparation procedure in action.

[7]This tagger can be obtained from `http://www.sejong.or.kr`.

Step	Data
Sentence	Wuntong-ul ha-myen wuli mom-eyse sanso-wa yengyangpun-i manhi sayongtoy-pnita.
Fragment	mom eyse yengyangpun i sayongtoy 'body LOC_2 nourishment NOM be-used'
Features	$j = \text{LOC}_2$, $p =$ sayongtoy, $n =$ mom, $d =$ far, $s =$ NOM $j =$ NOM, $p =$ sayongtoy, $n =$ yengyangpun, $d =$ near, $s = \text{LOC}_2$

Table 1: Training data preparation procedure in action

5.2 Evaluation

The performance of our disambiguation technique is measured against a test set which was annotated by two human annotators. The test set is composed of 550 instances. We randomly picked up 50 sentences that have case ambiguities for every 11 pre-selected predicates. These predicates were also used for evaluations in previous studies Kim (1996) and Chung (1999).

We report pairwise agreement of case decision between annotators and our method in percentage. We also show pairwise Kappa statistic to take account of expected chance inter-annotator agreement. Kappa enables us to measure the agreement factoring out the expected chance agreement (Cohen 1960).[8] The percentage agreement between our two annotators is 97.82% and the Kappa value is 5.38, which belongs to the moderate agreement range according to Landis and Koch (1977). Table 2 reveals that our two human annotators have disagreed on 'Reject' many times.[9] Only 6 out of 12 instances are agreed. Naturally this is reflected on the Kappa statistic.

Class	NOM	ACC	LOC_1	LOC_2	INST	Reject	Sum
NOM	360	2	1	0	0	0	363
ACC	2	159	0	0	0	0	161
LOC_1	1	0	9	0	0	0	10
LOC_2	0	0	0	2	0	0	2
INST	0	0	0	0	2	0	2
Reject	6	0	0	0	0	6	12
Sum	369	161	10	2	2	6	550

Table 2: Contingency table for two human annotators' responses

6 Result and Discussion

Table 2 shows the pairwise agreement in percentage and Kappa value between two statistical classifiers and two human judges K and Y. Pairwise agreements between two other simple models are also shown.

Human	$P(j)$	SOV	NB	BO
K	29.60 (1.61)	70.40 (3.83)	45.95 (2.50)	78.86 (4.29)
Y	29.93 (1.61)	70.26 (3.78)	46.47 (2.50)	79.74 (4.29)
AVG	29.77 (1.61)	70.33 (3.81)	46.21 (2.50)	79.40 (4.29)

Table 3: Pairwise agreements between classifiers and human annotators

[8] Refer to Siegel and N. John Castellan (1988) for the detailed description of calculation method.
[9] Annotators are requested to mark 'Reject' when a noun cannot be used with any case particle.

$P(j)$ is a case assignment model that always chooses the most frequent case particle *lul* ACCUSATIVE. This model yields 29.97% average agreement with human annotators. Kappa value is very low.

SOV is our baseline model which reflects the canonical word order in Korean: SUBJECT OBJECT VERB. This model selects a case particle according to the distance between the noun and the predicate in question. This model picks up *ka* NOMINATIVE for $d = far$, and *lul* ACCUSATIVE for $d = near$. The performance of this model is surprisingly good. Average agreement in percentage is 70.33%. Kappa is also far better than $P(j)$.

The average agreement and Kappa value between NB and human annotators are 46.21% and 3.83. These figures are far lower than those of SOV. The low performance of NB suggests that the combinations of feature values are crucial in our classification task.

BO[10] shows the best performance in our experiment. The average agreement with a human annotator in percentage reaches up to 79.40%. This shows the effectiveness of our back-off method.

Table 3 shows the back-off stages and the agreements between the stages and the response of a human annotator. According to this table, our model was able to cover most of the test instances at the triples stage.

Stage	No. of Inst	%	No. of Agree	%
Quadruples	134	24.72	117	87.97
Triples	412	74.54	310	77.31
Doubles	4	0.74	2	50.00
Singles	0	0.00	0	0.00
Default	0	0.00	0	0.00
Totals	538	100.00	429	79.74

Table 4: Back-off stages and the agreements with a human annotator Y

The agreement percentage is still below the reported accuracies of previous work. For example, Chung (1999), which uses class-based smoothing technique, reported 86.16% case assignment accuracy.[11] The testing material of current study and those of previous work are different in their size and construction method.[12] Therefore, direct comparison of performance is not possible.

Class	NOM	ACC	LOC_1	LOC_2	INST	Sum
NOM	273	32	48	7	3	363
ACC	5	148	7	0	1	161
LOC_1	3	0	7	0	0	10
LOC_2	1	0	1	0	0	2
INST	0	0	1	0	1	2
Sum	282	180	64	7	5	538

Table 5: Contingency table for BO (col) and a human annotator Y (row)

Table 4 is the contingency table for BO and a human annotator. This table shows that BO largely disagree with Y on assigning *ka* NOMINATIVE case.

[10]We obtained the best performance when we set the back-off cut point k to 5.

[11]We were unable to reimplement this model since we could not obtain the lexical resource used in the model at the time of writing.

[12]Some studies included instances from training examples into their testing data.

(2) a. Wuli-tul-uy sinmwun-un thanap *pat*-ass-ta.
we-PL-GEN news-paper-TOP oppression receive-PAST-DECL
'Our news papers were oppressed.'
Y: *ka* NOMINATIVE
BO: *lul* ACCUSATIVE

b. Cwunsiki-nun kulen misu yu-ekey wyngkhu-lul *ponay*-ess-ta.
Cwunsiki-TOP such miss Yu-DAT wink-ACC send-PAST-DECL
'Cwunsiki sent a wink to Miss Yu.' Y: *ka* NOMINATIVE
BO: *ey* LOCATIVE$_2$

c. kuke-n cey-ka *sse*-ss-supnita.
that-TOP I-NOM write-PAST-DECL
'I wrote that.'
Y: *lul* ACCUSATIVE
BO: *ka* NOMINATIVE

Sentences in (2) show confusions between *ka* NOMINATIVE and other case particles. For the noun *sinmwun* 'news paper' in (2a), annotator Y assigned *ka* NOMINATIVE while BO assigned *lul* ACCUSATIVE. This confusion can be attributed to the fact that the other noun in the sentence *thanap* 'oppression' occurs without any particle. Current model only sees the neighbouring case particles. If we can extend our model to use information from surrounding nouns, this confusion may be prevented.

(2b) is another example of *ka* NOMINATIVE confusion, where BO decided the case of *Cwunsiki* as *eyse* LOCATIVE$_2$. It is not trivial to deal with this sentence since we only have rough approximations of subcategorisation frames. We may need to find a way of learning more precise subcategorisation frames and a matching technique. Alternatively, it would be also desirable to incorporate the oblique or case hierarchy into the model. Hierarchical order information of cases could guide the model to choose the right target.

In (2c), *kuke* 'that' was assigned *ka* NOMINATIVE by BO. The actual case should be *lul* ACCUSATIVE. As already mentioned, the canonical word order of Korean is SUBJECT OBJECT VERB, and we are using this information for case assignment. This word order feature has lower priority than other features. This is one of the limitation of current model which uses a very simple feature combination method and treat the features equally when it backs-off.

Finally, we present a learning curve of our statistical model for case assignment in Korean in Figure 1.[13] The agreement value of BO radically increases in earlier stages. The slope of the curve reaches the second top score 79.55% at 500K point. After that it falls down to 77.14% and slowly goes up again reaching 79.73%. We can't easily predict how the shape of the curve will change if we use more training examples. As a whole, however, the agreement value keeps increasing as the number of training examples grows.

7 Conclusion and Future Work

In this paper, we proposed a very simple statistical model for the case assignment task in Korean. Experiment result and analysis reveal that our model can effectively assign cases for nouns used without case particles even if we feed the model a set of imperfect training instances extracted from an unannotated corpus using a very simple and cheap procedure.

We also identified the limitations of current model. Following is a list of issues we can tackle in future work to enhance our model.

[13]The agreement in percentage measured against the response of a human annotator Y.

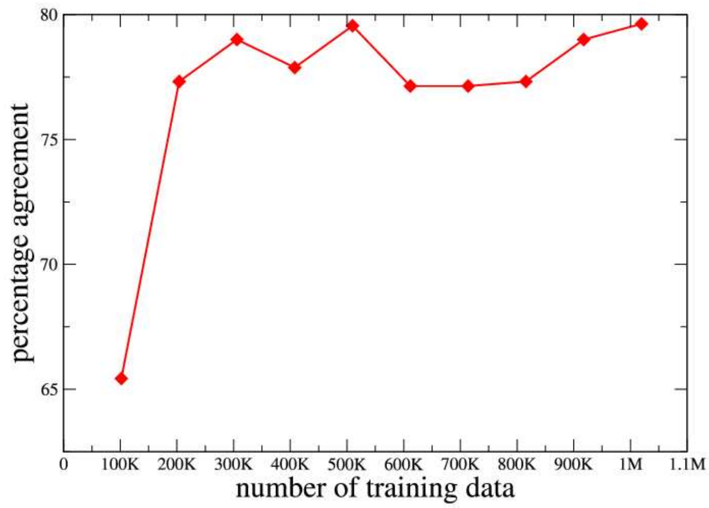


Figure 2: Learning curve of BO

- More features. It would be desirable to incorporate more features such as neighbouring nouns and other linguistic information like case hierarchy and detailed part-of-speech into the model.

- Other smoothing methods. If we incorporate more features into our model, we can try to use other smoothing/combination methods such as a linear interpolation.

- More training data. Figure 1 indicates that the agreement value keeps increasing as the number of training examples grows. The behaviour of our model could be very different with more training data (Banko and Brill 2001a,b). We can use the Web as a corpus (Keller and Lapata 2003).

- Class-based Smoothing. Word clustering (Li 2002) or LSA (Landauer et al. 1998) can be used for smoothing since a lexical resource comparable to WordNet in Korean is not easily accessible.

- Alternative training data collection method. Our training data collection method is very simple and conservative. If we use more aggressive techniques, we could use corpus more efficiently.

Acknowledgement

Many thanks to Henry S. Thompson and Miles Osborne for their endless support and encouragement. I am also grateful to the anonymous reviewers for the helpful comments.

References

Banko, Michele, and Eric Brill. 2001a. Mitigating the Paucity-of-Data Problem: Exploring the Effect of Training Corpus Size on Classifier Performance for Natural Language Processing. In *Proceedings of the Conference on Human Language Technology*.

Banko, Michele, and Eric Brill. 2001b. Scaling to Very Very Large Corpora for Natural Language Disambiguation. In *Proceedings of the 39th Annual Meeting and 10th Conference of the European Chapter (ACL-EACL '2001)*, 26–33.

Buchholz, Sabine, Jorn Veenstra, and Walter Daelemans. 1999. Cascaded Grammatical Relation Assignment. In *Proceedings of the Joint SIGDAT conference on Empirical Methods in Natural Language Processing and Very Large Corpora (EMNLP/VLC '99)*, 239–246.

Carroll, John, and Ted Briscoe. 2002). High Precision Extraction of Grammatical Relations. In *Proceedings of the 19th International Conference on Computational Linguistics (COLING '02)*, 134–140. Also available at `http://www.cogs.susx.ac.uk/lab/nlp/carroll/papers/coling02.pdf`.

Cho, Pyoungok, and Chyulhyung Ok. 1997. Construction of semantic hierachies of Korean Nouns. In *Proceedings of the 9th Conference on Hangul and Korean Information Processing*, 313–319. In Korean.

Chung, Hoojung. 1999. Resolving Syntactic Ambiguity Using Lexical Information. Master's thesis, Department of Computer Science, Korea University. In Korean.

Cohen, Jacob. 1960. A Coefficient of Agreement for Nominal Scales. *Educational and Psychological Measurement* 20: 37–46.

Collins, Michael J. 1999. A Head-Driven Approach to Statistial Natural Language Parsing. Ph.D. thesis, Department of Computer and Information Science, University of Pennsylvania.

Collins, Michael J., and James Brooks. 1995. Prepositional Phrase Attachment Through a Backed-off Model. In *Proceedings of the Third Workshop on Very Large Corpora*, 27–38.

de Lima, Erika F. 1997. Assigning Grammtical Relations with a Back-off Model. In *Proceedings of the 2nd Conference on Empirical Methods in Natural Language Processing (EMNLP-2)*. Also available as `http://xxx.soton.ac.uk/ps/cmp-lg/9706001`.

Duda, Richard O., and Peter E. Hart. 1973. *Pattern Classification and Scene Anlysis*. New York: Wiley.

Grefenstette, Gregory. 1997. SQLET: Short Query Linguistic Expansion Techniques: Palliating One or Two-word Queries by Providing Intermediate Structure to WWW Pages. In *Proceedings of the 5th Conference On Intelligent Multimedia Information Retrieval Systems and Management (RIAO '97)*, 500–509. Also available as `http://www.xrce.xerox.com/publis/mltt/mltt-97-10.ps`.

Hockenmaier, Julia. 2003. Data and Models for Statistical Parsing with Combinatory Categorial Grammar. Ph.D. thesis, School of Informatics, University of Edinburgh.

Katz, Slava M. 1987. Estimation of Probabilities from Sparse Data for the Language Model Compoment of a Speech Recogniser. *IEEE Transactions on Acoustics, Speech, and Signal Processing* 35(3): 400–401.

Keller, Frank, and Mirella Lapata. 2003. Using the Web to Obtain Frequencies for Unseen Bigrams. *Computational Linguistics* .

Kim, Heungkyu. 2001. Korean Language Data Construction. Research report, National Academy of the Korean Language. In Korean.

Kim, Seon Ho. 1996. A Prediction of Lexical Relation Based on Statistical Information. Master's thesis, Department of Computer Science, Yonsei University. In Korean.

Lafferty, John, Daniel Sleator, and Davy Temperley. 1992. Grammatical Trigrams: A Probabilistic Model of Link Grammar. In *Proceedings of the AAAI Conference on Probabilistic*

Approaches to Natural Language, 89–97. Also available as http://www.cs.cmu.edu/afs/cs.cmu.edu/project/link/pub/www/papers/ps/gra%m3gram.ps.

Landauer, T. K., P. W. Foltz, and D. Laham. 1998. Introduction to Latent Semantic Analysis. *Discourse Processes* 25: 259–284.

Landis, J. Richard, and G. G. Koch. 1977. The Measurement of Observer Agreement for Categorial Data. *Biometrics* 33: 159–174.

Lee, Iksop, and S. Robert Ramsey. 2000. *The Korean Language*. Albany: State Univerity of New York Press.

Li, Hang. 2002. Word Clustering and Disambiguation based on Co-occurrence Data. *Natural Language Engineering* 8(1): 25–42. Also available at http://research.microsoft.com/users/hangli/HP_files/Li-NLE02.pdf.

Li, Hui-Feng, Jong-Hyeok Lee, and Geunbae Lee. 1998. Identifying Syntactic Role of Antecedent in Korean Relative Clause using Corpus and Thesaurus Information. In *Proceedings of the 17th International Conference on Computational Linguistics and the 36th Annual Meeting of the Association for Computational Linguistics (COLING-ACL '98)*, 756–762.

Lim, Hongbin. 1993. A study on thesaurus of Korean words. Research report, National Academy of the Korean Language. In Korean.

Mitchell, Tom. 1997. *Machine Learning*. McGraw-Hill.

Ohno, S., and M. Hamanishi. 1981. *New Synonym Dictionary*. Tokyo: Kadokawa Shoten. In Japanese.

Palmer, Martha, Rebecca Passonneau, Carl Weir, and Tim Finin. 1993. The KERNEL Text Understanding System. *Artificial Intelligence (Special Issue on Text Understanding* 63: 17–78. Also available as http://www.cis.upenn.edu/~mpalmer/papers/kernel.ps.gz.

Ratnaparkhi, Adwait. 1998. Statistical Models for Unsupervised Prepositional Phrase Attachment. In *Proceedings of the 17th International Conference on Computational Linguistics and the 36th Annual Meeting of the Association for Computational Linguistics (COLING-ACL '98)*, 1079–1085.

Resnik, Phillip. 1993. Selection and Information: A Class-based Approach to Lexical Relations. Ph.D. thesis, University of Pennsylvania.

Seo, Sangkyu. 1999. Computer as a Tool of Linguistic Research. *Research on Language and Information* 1: 271–309. In Korean.

Siegel, Sidney, and Jr. N. John Castellan. 1988. *Nonparametric Statistics for the Behavioural Science (2nd ed)*. New York: McGraw-Hill.

Sohn, Ho-Min. 1999. *The Korean Language*. Cambridge Language Surveys. Cambridge University Press.

Yang, Jaehyung, and Yung Taek Kim. 1993. Identifying Deep Grammatical Relations in Korean Relative Clauses Using Corpus Information. In *Proceedings of Natural Language Processing Pacific Rim Symposium '93 (NLPRS '93)*, 337–344.

Yang, Jaehyung, and Yung Taek Kim. 1994. A Statistical Approach to the Resolution of Ambiguous Grammatical Functions In Korean Noun Phrases. *The Journal of the Korea Information Science Society* 21(5): 808–815. In Korean.

Yeh, Alexander. 2000. Using Existing Systems to Supplement Small Amounts of Annotated Grammatical Relations Training Data. In *Proceedings of the 38th Annual Meeting of the Association for Computational Linguistics (ACL '2000)*, 126–132. Also available as http://xxx.soton.ac.uk/ps/cs.CL/0010020.

Aggregative combinatorics: An introduction

Tara Nicholson
Laboratory for Logic & Experimental Philosophy
Department of Philosophy
Simon Fraser University
Burnaby, BC, Canada V5A 1S6
taraleen@sfu.ca

Martin Allen
Computer Science Department
University of Massachusetts
Amherst, MA 01003
mwallen@cs.umass.edu

Abstract

This paper is an exploratory introduction to issues surrounding the combinatorial expressivity and computational complexity of weakly aggregative modal logic (**WAL**) – the latter insofar as it relates to the former, but also more generally. We begin by showing how WAL allows us to define a natural hierarchy of (recognition) optimization problems not generally available in logics as strong as K. In this respect we illustrate that an important advantage of the *Aggregative Programme* in modal logic (as dubbed by its progenitors, Jennings and Schotch) over the standard modal paradigm, which views K as the weakest normal modal logic, has to do with its expressive, discriminatory power. As an extended example in this context we formulate the Four Colour Theorem (Every planar graph is 4-colourable) (**4CT**) as a rule schema in WAL which preserves validity with respect to quinary relational frames if and only if 4CT is true. Accordingly, because WAL is decidable, additional motivations for exploring its combinatorial expressivity arise from issues of decidability in graph theory. In closing we analyze the complexity of the satisfiability problem for these logics.

1 Introduction

Whereas the standard (Kripkean) approach to modal logic views K as the base of the lattice of inclusion according to which all normal modal logics are ordered (Schotch and Jennings, 1980), on the *Aggregative Programme* in modal logic, this paradigm is inverted to accommodate the study of logics weaker than K (Jennings and Schotch, 1981). First espoused in detail in (Schotch and Jennings, 1980), motivations for this programme have included (a) its ability to discern semantical limitations of the standard approach, particularly with respect to first-order (in)definability in modal logic,[1] and the fact that, (b) like the logics K and E, determined by the class of Scott-Montague, or neighborhood, and binary relational frames, respectively, the class of $\boldsymbol{K_n}$ **logics** ($n \in \mathbb{Z}+$), with which aggregativity theory is mainly concerned, is also determined by an unrestricted class of structures (Jennings and Schotch, 1981). Specifically, *weakly aggregative logics*, as they are also called, are determined by the class of $(n+1)$-ary relational frames, their alias borrowed from a natural combinatorial property modelled by these structures, given the truth-condition defined for the necessity operator.

We begin with a survey of the basic ideas and results associated with these logics, before proceeding to our new results.

Definition 1.1. *Let U be a non-empty set, and let $R \subseteq U^{n+1}$. Then $\mathfrak{F} = (U, R)$ is an* $\boldsymbol{(n+1)}$***-ary relational frame*** *('frame', for short). Further, if $\mathfrak{F}$ is a frame and V is a valuation function from propositional variables to subsets of U which is defined as usual for Boolean operators and extended to modal formulae via:*

$$\mathbf{V}(\Box\mathbf{A}) \;:=\; \{x \in U \mid \forall (y_1, ..., y_n) \in U^n, Rxy_1, ..., y_n \Rightarrow \exists i \; (1 \leq i \leq n) : y_i \in V(A)\}, \tag{1.1}$$

then the pair $(\mathfrak{F}, V)$ is an $(n+1)$-ary relational ***model*** $\boldsymbol{\mathfrak{M}}$ *(on $\mathfrak{F}$).*

[1] For instance, Jennings et al. have shown that neither $[D] : \Box p \rightarrow \diamond p$ nor $[G] : \Diamond\Box p \rightarrow \Box\Diamond p$ is first-order definable using only a single n-ary predicate ($n \geq 3$) (Jennings, Schotch, and Johnston, 1980), (Jennings, Schotch, and Johnston, 1981), (Schotch, Jennings, and Johnston,), notwithstanding that both formulae are first-order definable using a single binary predicate.

In proof-theoretic terms, this combinatorial property can be seen as the condition that for each $n > 1$, any complete axiomatization of the logic determined by the class of frames is distinguished from a complete axiomatization of its predecessor by a *weakening* of the extent to which individually necessary formulae are jointly necessary. In the system K_n this distinguishing condition takes the following form:

$$[K_n]: \quad \vdash \Box p_1 \wedge ... \wedge \Box p_{n+1} \to \Box \bigvee_{i,j\in[n+1](i\neq j)} p_i \wedge p_j,$$

which, enjoined with:

$$\begin{aligned} [RR]: &\quad \vdash A \to B \;\Rightarrow \vdash \Box A \to \Box B, \\ [RN]: &\quad \vdash A \;\Rightarrow \vdash \Box A, \text{ and} \\ [RPL]: &\quad \vdash_{PL} A \;\Rightarrow \vdash A, \end{aligned}$$

constitutes a sound and complete axiomatization of the logics of $(n+1)$-ary frames (Apostoli and Brown, 1995), (Nicholson, Jennings, and Sarenac, 2000), (Urquhart, 1995).[2] Thus, using the idiom of aggregativity, the system N, consisting solely of $[RR]$,$[RN]$, and $[RPL]$(see (Jennings and Schotch, 1981)), can be said to be *non-aggregative*, and the system $K(= K_1)$, *maximally aggregative*. In fact, it is proved in (Jennings and Schotch, 1981) that the system N is the intersection of the denumerable sequence of K_n systems, of which the limit is, to use the standard idiom, the 'minimal' logic, K. Note that these logics, while weak with respect to aggregation, are in other respects typical modal logics; that is, they are normal, due to the presence of $[RN]$, and the relation $\vdash$ has all the properties, such as monotonicity and transitivity, usually associated with a consequence-relation.

In this paper, we consider yet another limitation of the standard modal paradigm, viz., that it obfuscates natural connections between relational modal semantics and graph colouring problems. For example, the perspective which views K as the weakest logic fails to see that the validity of $[K]$ $(= [K_1])$ (on binary frames) is equivalent to the 1-uncolourability of the complete graph on two vertices, $\mathbb{K}_2$.[3] Similarly occluded is the fact that completeness for K is essentially a consequence of the sufficiency of binary conjunction for the logical formulation of any 1-uncolourable graph (Nicholson, Jennings, and Sarenac, 2000). More generally it is possible to show that for each $n \geq 1$,

1. the soundness of K_n is equivalent to the n-uncolourability of $\mathbb{K}_{n+1}$, and

2. the completeness of K_n is equivalent to the functional completeness of [RPL] enjoined with the truth function $\frac{2}{n+1}$ $(= \bigvee_{i,j\in[n+1](i\neq j)} p_i \wedge p_j)$, with respect to the derivation of all (uncolouring) formulations of n-uncolourable graphs (Nicholson, Jennings, and Sarenac, 2000), (Nicholson, Jennings, and Sarenac, 2001).

We don't go into details regarding these specific facts here; the reader is directed to the associated references. Instead, we focus on the following new results:

I. the ability of K_n to represent problems from various branches of combinatorics, and mathematics more generally – specifically, those in which the Four Colour Theorem finds equivalent expression (see (Saaty, 1972) for an extensive list), and

II. the expressivity of K_n with respect to structural relationships between distinct combinatorial problems – specifically, optimization problems involving graph colourings.

In the epilogue we show that:

III. the satisfiability problem for K_n is PSPACE-complete in general, and NP-complete for formulae of bounded modal operator depth.

The paper is structured as follows: (II) is presented in Section 2, and (I) in Section 3; Sections 4 and 5 address (III) and a related conjecture of Vardi (Vardi, 1989).

[2] A dual axiomatization can be found in (Nicholson, Jennings, and Sarenac, 2001).

[3] We use the notation '$\mathbb{K}_m$' to denote the isomorphism class of complete graphs on m vertices $(m > 0)$. Since it is standard practice among graph-theorists to identify this class with any of its elements, we may also speak of *the* complete graph $\mathbb{K}_m$ on m vertices, without breaking with convention.

2 The Colouring Fragment

One reason for interest in weakly aggregative modal logics is that they allow us to define a natural class of combinatorial problems not generally available in logics as strong as K. Essentially this is because the language of aggregativity can be equivalently recast in the language of optimization problems whose solutions involve a determination of (un)colourability properties of graphs. Inasmuch as an optimization problem $\mathcal{P}$ can be seen as a reasoning problem whose domain is the solution space of an instance of $\mathcal{P}$ (Wong, 2003), differing degrees of aggregativity can therefore be seen as determining differing degrees of expressive power with respect to the representation of a class of spatial reasoning problems. To illustrate this fact, we show how K_n distinguishes between the elements of a denumerable class of recognition tasks.

For our purposes it is sufficient to understand a **graph G** as a pair (V, E) where the **node** or **vertex set V** of G (or **VG**) is a finite non-empty set, and the **edge set E** (or **EG**) is a finite non-empty set of pairs $e_i = \{v, w\}$, $(v \neq w)$ such that $\bigcup_{i=1}^{|E|} e_i = V$.[4] A graph G is ***m*-uncolourable** ($m \in \mathbb{Z}^+$) if there is no partition of VG into m pairwise disjoint, jointly exhaustive, cells such that every edge of G has a non-empty intersection with at least two cells; otherwise, G is ***m*-colourable**, and any such partition is an ***m*-colouring** of G.

In order to show how the aggregativity of the K_n logics is related to graph colouring, we need some way of treating graphs as modal sentences. To this end we use a standard, simple modal language $\mathcal{L}$ that has a denumerable set $\phi = \{p_1, p_2, ..., p_i, ...\}$ of propositional variables, where sub(super)scripts may be added as required.[5] Letting the letter p range over ϕ, a sentence (formula) A of the language $\mathcal{L}$ is defined:

$$A ::= p \mid \bot \mid \neg B \mid B \vee C \mid \Box B,$$

where $\rightarrow$, $\wedge$, $\Diamond$, *etc.* acquire their usual abbreviational status, and Φ is the set of all such formulae. Then where $G = ([j], E)$ is a graph such that $[j] \subseteq \phi$, the **formulation** of G, $\mathbf{F(G)}$ is defined:

$$\mathbf{F(G)} \ := \ \bigvee_{e \in E} \bigwedge_{f,h \in e} f \wedge h, \tag{2.1}$$

and the **uncolouring formulation** of G, $\mathbf{UNCOL}(\boldsymbol{G})$, is set:

$$\mathbf{UNCOL(G)} \ := \ \Box 1 \wedge ... \wedge \Box j \rightarrow \Box F(G). \tag{2.2}$$

Introducing the convention that for any sentence A and positive integer n, A is $\boldsymbol{(n+1)}$**-valid** iff for every model $\mathfrak{M}$ on any $(n+1)$-ary frame $\mathfrak{F} = (U, R), \forall x \in U, x \in V(A)$, it follows that:

Theorem 2.1. *$\forall n \geq 1, G = (V, E)$, UNCOL$(G)$ is $(n+1)$-valid $\Leftrightarrow$ G is n-uncolourable.*

We can then define the set $\mathbf{COLR} \subseteq \Phi$:

$$\mathbf{COLR} \ := \ \{A \mid \exists G : A = \neg UNCOL(G)\}. \tag{2.3}$$

Clearly, any formula $A \in \mathbf{COLR}$ is true at a point in a model, i.e., is $\boldsymbol{K_n}$**-satisfiable**, iff the corresponding graph G is n-colourable. Further, for each $n \geq 1$, it follows that the set:

$$\mathbf{COLR_{K_n}} = \{A \mid A \in COLR \ \& \ A \text{ is } K_n\text{-satisfiable}\}$$

is a proper subset of the same set defined for logic K_{n+1}. That is, we have:

Proposition 2.2. $\forall n$, $COLR_{K_n} \subset COLR_{K_{n+1}}$.

[4] Accordingly it is sometimes useful to identify a graph G with its edge set E, essentially treating G as a set of pairs. Note also that edges are unordered sets of distinct elements. Thus, our graphs are undirected, and contain no self-loops. We treat edges as sets in order to keep our definition a general as possible; we are working in the context of the theory of *hypergraphs*, where edge-sets can consist of tuples larger than pairs. This is also why we say "at least two cells" rather than "exactly two cells" in the next sentence; in a hypergraph, a proper colouring has to colour at least two elements of each edge-set differently. See (Nicholson, Jennings, and Sarenac, 2001) for details.

[5] We will often use the index of a variable as an abbreviation for it (e.g., p_i is abbreviated 'i'), and more generally, if V is a set with j elements, we sometimes use the notation '$[j]$'(=$\{1, 2, ..., j\}$) to refer to V, so that $[j]$ is intended to represent $|V|$.

In this way K_n is able to preserve distinctions relative to a denumerable class of optimization problems – distinctions, moreover, to which K is blind. To illustrate, let **CHRM NMBR** be the optimization problem which has as an instance I_G, for any graph G, the pair (F, c), where F, the set of feasible solutions for I_G, is the set of n-colourings of G ($n \in \mathbb{Z}^+$), and $c : F \to \mathbb{R}$ is a cost function satisfying the condition that $\forall f \in F, c(f) = |f|$; that is, the cost of a colouring is simply the number of colours used.[6] Then our definition of a graph entails that for any graph G, there is no feasible solution f of cost $c(f) \leq 1$ for instance I_G.[7] Whence, in fact, that $K(= K_1)$ is maximally aggregative; equivalently, the solution space F of the recognition version "Is there $f \in F$ such that $c(f) \leq r$?" of CHRM NMBR, when r is restricted to 1, is empty. But, plainly, the K-unsatisfiability of a formula A does not imply its K_n-unsatisfiability for every $n > 1$.[8] Indeed, it follows from our definition of a graph, particularly the fact that graphs contain no self-loops, that for every graph G there is an instance $I_G = (F, c)$ of CHRM NMBR such that for some $r > 1, \exists f \in F : c(f) \leq r$. But as any such f is easily transformed to a satisfying $(r+1)$-ary model $\mathfrak{M}$ where $|U|$ is bounded above by $r + 1$ (and conversely), it follows that the complexity of determining membership in $COLR_{K_n}$ ($n \geq 1$) is (polynomially) equivalent to that of determining membership in F, where r is restricted to n. In sum, we therefore have:

Theorem 2.3. *Each logic K_n has associated with it a distinct decision problem, corresponding to membership in $COLR_{K_n}$ for each $n \in \mathbb{Z}^+$, where in particular:*

$$\begin{aligned} COLR_K = COLR_{K_1} &= \emptyset. \\ COLR_{K_2} &\in P. \\ \forall n > 2, COLR_{K_n} &\textit{ is NP-complete.} \end{aligned}$$

3 A Modal Formulation of 4CT

3.1 Motivations

The history of the Four Colour Theorem (Every planar graph is 4-colourable[9]) (**4CT**) provides somewhat of a unique, computationally oriented, impetus for exploring logical representations of this, and other, propositions of graph theory. While it is generally acknowledged today to be true, the relatively short history of 4CT as a problem has been fraught with false promises in the form of fallacious 'proofs' (see (Thomas, 1998) for a survey). Although Appel and Haken (A&H) produced a proof in 1976 (Appel and Haken, 1977), (Appel, Haken, and Koch, 1977), that was greatly simplified by Roberston et al. in the late 1990s (Roberston et al., 1996), its validity has not been accepted without controversy, essentially because, as the authors of the more recent proof note, "(i) part of the … proof uses a computer, and cannot be verified by hand, and (ii) even the part of the proof that is supposed to be checked by hand is extraordinarily complicated and tedious, and as far as we know, no one has made a complete independent check of it" (Roberston et al., 1996). To remedy (ii) Roberston et al simplify matters insofar as they produce a version of that particular part of the proof (viz., "unavoidability") which "can be checked by hand in a few months, or, using a computer, … can be verified in a few minutes" (Roberston et al., 1996). However, with respect to (i), their proof remains as controversial as A&H's.

But if 4CT is representable as a *sentence* of K_n, or any decidable system,[10] this would imply the existence of an effective method for demonstrating its truth (or falsity) – small comfort, of course, since it wouldn't follow that any such method is feasibly discoverable or usable. Moreover, one

[6]Note that it would be sufficient to define cost-function c as operating from F to $\mathbb{Z}^+$, since this is the range of values for the the number of colours n. We retain the mapping to the reals to cohere to the definition of a cost function in the general definition of an optimization problem.

[7]If we allow graphs to have empty edge sets then this is false, since any such graph is 1-colourable. We have elected to adopt the definition which forbids empty edge sets for illustrative purposes in this context, and notational convenience elsewhere. It is easily checked that no significant difference results with respect to our main theorem (2.3) in this section, since on both definitions the decision task corresponding to $COLR_{K_1}$ is more or less trivially a member of complexity class P.

[8]This fact has been exploited in the development of a paraconsistent inference relation which is derivable from the closure conditions on the set of $\Box$ formulae true at a point in a model (Jennings and Schotch, 1980) (Schotch and Jennings, 1989).

[9]Contrapositively, if a graph G is 4-uncolourable, then G cannot be embedded in the plane without edge crossings.

[10]Decidability for K_n can be proved by applying the filtration method discussed in (Chellas, 1980).

might wish to argue that the latest proof of 4CT counts more than amply with respect to answering questions which normally motivate the desire to show decidability, especially as it has furnished us with a quadratic time algorithm for 4-colouring planar graphs (Roberston et al., 1996). Nevertheless, the question remains as to whether there is a non-constructive proof for the existence of an effective method, and also one that can be adapted to provide decidability results for other propositions of graph-theory. Given the tight connection between K_n and colouring problems, culminating in Theorem 2.3, a natural extension of the Aggregative Programme in modal logic therefore consists of an exploration of the extent to which 4CT, among other graph-related statements, is representable in K_n.

We initiate this exploration below. Our new contribution to the literature on 4CT is to show how to formulate a modal rule schema, [4ct] which universally preserves 5-validity iff 4CT is true. In this way we also show that K_n can be used to represent an expressivity requirement for any language functionally complete with respect to the graphic representation of optimization problems. To illustrate, suppose that $\mathfrak{L}$, a language for the graphic representation of optimization problems, does not distinguish between edge-crossings and nodes. For example, suppose that $\mathfrak{L}$ represents nodes of a graph G by line-crossings in a planar embedding of G. Then, if the schema [4ct] universally preserves 5-validity, $\mathfrak{L}$ cannot unambiguously represent any optimization problem $\mathcal{P}$ with graphic representation G, for any G whose uncolouring formulation is 5-valid.

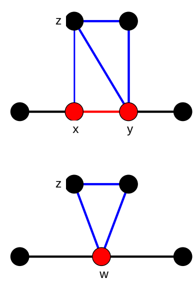


Figure 3.1.1: The graph on the bottom is the result of contracting the edge (x, y) in the graph on the top. That is, (x, y) is replaced by the node w, where w is joined by an edge to every node to which either x or y is joined. Multiple edges and loops are deleted.

3.2 Forbidden Minors

A graph H is a **forbidden minor** of a graph G iff H is isomorphic to either $\mathbb{K}_5$ or $\mathbb{K}_{3,3}$,[11] and, a graph H is a **minor** of a graph G, written '$\boldsymbol{G \succ H}$', if H is a subgraph of a graph obtainable from G by a finite sequence of edge contractions (see Figure 3.1.1). Moreover, it's not hard to see that if H has k nodes then $G \succ H$ iff G has k vertex-disjoint connected subgraphs $G_1, G_2, ..., G_k$ such that for each edge (v_i, v_j) in H, there is an edge (v_q, v_r) of G with $v_q \in VG_i$ and $v_r \in VG_j$ (Bollobás, 1991). Using this fact, our strategy is to formulate for every connected graph G, a non-trivial sentence schema $K(G)(= K_5(G) \vee K_{3,3}(G))$ such that $K(G)$ is 5-valid iff G has a forbidden minor.[12]

The phrase "non-trivial sentence schema" here is intended to convey two important facts. First, we give a *schema* in that for every pair of distinct graphs G and G', $K(G)$ and $K(G')$ differ at most in the encoding the mapping K imparts to their respective adjacency relations EG and EG'. Second, it is *non-trivial* in that, given Wagner's[13] characterization of planarity (Wagner, 1937):

$$\forall G, G \text{ is planar iff } G \text{ has no forbidden minor,} \tag{3.1}$$

[11] $\mathbb{K}_{3,3}$ is the complete bipartite graph on node sets of size 3.

[12] In what follows we restrict ourselves to connected graphs. For any graph G, if H is a connected graph then $G \succ H$ implies that there is a connected subgraph G' of G such that $G' \succ H$. Thus, since both $\mathbb{K}_5$ and $\mathbb{K}_{3,3}$ are connected, our sentence schema applies at least to some subgraph of any graph having one of these as a minor.

[13] And independently, Harary and Tutte's (Harary and Tutte, 1965).

we have that a proof that the modal rule schema [4ct]:

$$\frac{\text{UNCOL}(G)}{K(G)} \tag{3.2}$$

preserves 5-validity for every graph G is therefore also a proof of 4CT (and conversely). While [4ct] has not yet been shown to preserve 5-validity, its existence opens up a new avenue in the search for an alternative, non-constructive proof of 4CT.

3.2.1 The schema $K_m(G)$

For the sake of generality, we formulate a schema $K_m(G)$ such that $\forall G, m > 1, G \succ \mathbb{K}_m \Leftrightarrow K_m(G)$ is m-valid.[14] If G is a graph, with $m \in \mathbb{Z}^+$, then $\mathbb{G}^{\boldsymbol{m}} = \boldsymbol{\{\pi_1, \pi_2, ..., \pi_q, ..., \pi_r\}}$ is the set of all m-partitions $\pi_q = \{G_1^q, G_2^q, ..., G_m^q\}$ of G into pairwise vertex-disjoint connected subgraphs of G. And, if G is a graph with a minor M such that $VM = \pi_q \in \mathbb{G}^m$, then $\boldsymbol{G[\pi_q]}$ is the set of pairs $(u_s^{q_i}, v_t^{q_j})$ $(i \neq j)$ satisfying:

$$s \in [VG_i^q],\ t \in [VG_j^q], \tag{3.3}$$

$$u \in VG_i^q,\ v \in VG_j^q, \text{ and} \tag{3.4}$$

$$\exists e = (u, v) \in EG. \tag{3.5}$$

In other words, $G[\pi_q]$ is the set of G-edges, none of which occurs in any one subgraph in π_q, and each of whose nodes is appropriately labelled. Then if G is a graph, $m \leq |VG|$, and $\pi_q = \{G_1^q, G_2^q, ..., G_i^q, ..., G_m^q\}$ is in $\mathbb{G}^m$, the schemas $\boldsymbol{M^{\pi_q}(G)}$, and $\boldsymbol{K_m(G)}$ are defined:

$$\mathbf{M}^{\pi_\mathbf{q}}(\mathbf{G}) \ := \ \bigwedge_{i=1}^{m} \Box \bigwedge_{s=1}^{[VG_i^q]} u_s^{q_i} \rightarrow .\ \Box F(G[\pi_q]). \tag{3.6}$$

$$\mathbf{K_m}(\mathbf{G}) \ := \ \bigvee_{q=1}^{r} M^{\pi_q}(G). \tag{3.7}$$

3.2.2 The schema $K_{3,3}(G)$

By forcing models to ignore 4-colourings of $\mathbb{K}_{3,3}$, the aggregative tendencies of formulae on quinary frames can be exploited to adapt the strategy used in formulating $K_m(G)$ to the present case.[15]

If G is a graph, then let $\mathbb{G}^{\mathbf{3,3}}$ be the set $\{\varpi_1, \varpi_2, ..., \varpi_q, ..., \varpi_r\}$ of all pairs $\varpi_q = \{\mathcal{D}_1^q, \mathcal{D}_2^q\}$ that satisfy the following conditions:

$$\exists G_1, G_2 \subseteq G \text{ such that } \mathcal{D}_1^q \in \mathbb{G}_1^3, \mathcal{D}_2^q \in \mathbb{G}_2^3, \text{ and} \tag{3.8}$$

$$VG_1 \cap VG_2 = \emptyset, \text{ and } VG_1 \cup VG_2 = VG. \tag{3.9}$$

In other words, ϖ_q is a partition of G into two distinct 3-partitions, $\mathcal{D}_1^q = \{G_1^{q_1}, G_2^{q_1}, G_3^{q_1}\}$ and $\mathcal{D}_2^q = \{G_1^{q_2}, G_2^{q_2}, G_3^{q_2}\}$, of vertex-disjoint subgraphs G_1 and G_2 of G, respectively. Evidently then, a graph G has the forbidden minor $\mathbb{K}_{3,3}$ if and only if such a partition exists, with edges between the cells of the partition corresponding to the relevant edges of $\mathbb{K}_{3,3}$. That is, we have:

Proposition 3.1. *$\forall G, G \succ \mathbb{K}_{3,3}$ iff there exists some ϖ_q such that $\forall i, j \in [3]$, $\exists e = (s,t) \in EG$ with $s \in VG_i^{q_1}$, and $t \in VG_j^{q_2}$.*

[14] The reader familiar with **Hadwiger's Conjecture** ($\forall G, n \in \mathbb{Z}^+$, if G is n-uncolourable then $G \succ K_{n+1}$) will perceive that we can prove as a corollary that Hadwiger's Conjecture is true iff for any G, the rule schema $\frac{UNCOL(G)0}{K_{n+1}(G)}$ preserves $(n+1)$-validity.

[15] A similar strategy is used in (Nicholson, 2001) to develop a construction with applications towards a possible proof of Hadwiger's Conjecture.

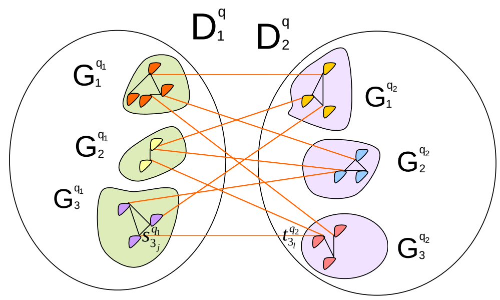


Figure 3.2.1: A graph G with $\varpi_q = \{D_1^q, D_2^q\} \in \mathbb{G}^{3,3}$. The edges (in orange) spanning the two largest circles comprise the edge set of the graph $G(D_1^q, D_2^q)$.

So, to represent any edge (s,t) in a potential $\mathbb{K}_{3,3}$ minor of G (relative to ϖ_q), we define $\boldsymbol{G(D_1^q, D_2^q)}$ (see Figure 3.2.1) as the set of pairs $(s_{i_j}^{q_1}, t_{k_l}^{q_2})$ satisfying:

$$i, k \in [3], \tag{3.10}$$
$$j \in [VG_i^{q_1}], l \in [VG_k^{q_2}], \tag{3.11}$$
$$s \in VG_i^{q_1}, t \in VG_k^{q_2}, \text{ and} \tag{3.12}$$
$$e = (s,t) \in EG. \tag{3.13}$$

Then the schemas $\boldsymbol{M^{(D_1^q, D_2^q)}(G)}$ and $\boldsymbol{K_{3,3}(G)}$ are defined as:

$$\begin{aligned}\mathbf{M^{(D_1^q, D_2^q)}(G)} \;&:=\; \bigwedge_{i=1}^{3} \Box \bigwedge_{h=1}^{[VG_i^{q_1}]} s_{i_h}^{q_1} \;\wedge\; \bigwedge_{i=1}^{3} \Box \bigwedge_{h=1}^{[VG_i^{q_2}]} t_{i_h}^{q_2} \;\wedge \\ &\quad \bigvee_{i=1}^{3} \bigvee_{j=1}^{3} \Box \Big(\bigwedge_{h=1}^{[VG_i^{q_1}]} s_{i_h}^{q_1} \wedge \bigwedge_{h'=1}^{[VG_j^{q_2}]} t_{j_{h'}}^{q_2} \Big) \rightarrow . \;\; \Box F(G(D_1^q, D_2^q)).\end{aligned} \tag{3.14}$$

$$\mathbf{K_{3,3}(G)} \;:=\; \bigvee_{q=1}^{r} M^{(D_1^q, D_2^q)}(G). \tag{3.15}$$

3.2.3 The schema $K(G)$

In the interest of satisfying restrictions imposed on the length of this paper, we establish our central result pertaining to 4CT, viz.,

Theorem 3.2. *$\forall G$, $\frac{UNCOL(G)}{K(G)}$ preserves 5-validity iff 4CT is true.*

For purposes of proof, we sketch the 'hard' direction of the following lemma:

Lemma 3.3. *$\forall G$, $K(G)$ is 5-valid iff G is not planar,*

where for any graph G, the schema $\boldsymbol{K(G)}$ is defined as:

$$\mathbf{K(G)} \;:=\; K_{3,3}(G) \vee K_5(G). \tag{3.16}$$

Proof. Let G be a planar graph. Then by Wagner's theorem (3.1), $G \not\succ K_{3,3}$ and $G \not\succ K_5$. Thus, from Proposition 3.1 it follows that $\forall \varpi_q = \{D_1^q, D_2^q\} \in \mathbb{G}^{3,3}$, $\exists i, j \in [3]$ such that no $e \in EG$ has endpoints in both $G_i^{q_1}$ and $G_j^{q_2}$. Consequently there is a 4-partition $\pi'_q = \{c_1^q, ..., c_4^q\}$ satisfying:

$$\forall h \in [3], \exists c^q \in \pi'_q : VG_h^{q_1} \subseteq c^q, \tag{3.17}$$
$$\forall h \in [3], \exists c^q \in \pi'_q : VG_h^{q_2} \subseteq c^q, \tag{3.18}$$
$$\exists c^q \in \pi'_q, i, j \in [3] : VG_i^{q_1} \cup VG_j^{q_2} \subseteq c^q, \text{ and} \tag{3.19}$$
$$\forall e \in G(D_1^q, D_2^q), \forall c^q \in \pi'_q, e \not\subseteq c^q. \tag{3.20}$$

We use $\pi'_q = \{c^q_1, ..., c^q_4\}$ to define a model $\mathfrak{M} = (\mathfrak{F}, V)$, on a quinary frame $\mathfrak{F} = (U, R)$ such that for some $x \in U$, $x \notin V(K_{3,3}(G))$. To this end let π^* be the set $\{c^*_i \mid i \in [4]$ and $c^*_i = \bigcup_{q=1}^{r} c^q_i\}$. Then where:

$$U := \pi^*(= \{c^*_1, ..., c^*_4\}) \cup \{x\}, \text{ for some new variable } x, \tag{3.21}$$
$$R := \{(x, c^*_1, ..., c^*_4)\}, \text{ and} \tag{3.22}$$
$$\forall p \in \phi, c^*_i \in V(p) \text{ iff } p \in c^*_i, \tag{3.23}$$

it is straightforward to show that $x \notin V(K_{3,3}(G))$. By similar reasoning, the model $\mathfrak{M}$ can be augmented in such a way that $K_5(G)$ is simultaneously false at x. Whence $K(G)$ is not 5-valid if G is planar. $\square$

4 The complexity of K_n

Ladner (Ladner, 1977) has shown that the satisfiability question for any modal logic S such that $K \leq S \leq S4$ is PSPACE-complete.[16] The proof relies upon a reduction from the satisfiability problem for QBFs (Quantified Boolean Formulae), known to be PSPACE-complete (Stockmeyer and Meyer, 1973). For any QBF A of the form $Q_1p_1Q_2p_2 \dots Q_mp_mA'$, where each Q_i is either $\forall$ or $\exists$, and A' is a quantifier-free formula containing all and only propositional letters $p_1, \dots, p_m$, we can derive a modal formula B such that B is satisfiable in the modal logic S iff A is a satisfiable QBF. To do so, we define B so that a modal tree-structure satisfying it has leaves that mimic an appropriate set of truth-assignments to the propositional letters in A. Halpern (Halpern, 1995) has shown that the PSPACE bound is robust under various restrictions on the number of propositional letters appearing in modal formula B. At the same time, he shows that bounding the maximum *depth* of modal formulae, measured in terms of the nesting of modal operators, can in some cases reduce the complexity of modal SAT. In particular, for formulae of depth $< k$, $k \in \mathbb{N}$, the satisfiability problem for logics K and T is NP-complete; for $S4$, the problem remains PSPACE-complete.[17] We generalize these established results, showing for the first time that they extend to logics K_n.

4.1 K_n-SAT

Due to the weakly-aggregative nature of logics K_n, $n \geq 2$, the Ladner/Halpern proofs of PSPACE bounds on satisfiability do not immediately go through, since these proofs rely upon the power of the $\Box$ operator to force various formulae to come out true in single worlds along a modal tree-structure. For any logic weaker than $K = K_1$, however, the presence of two formulae $\Box A$ and $\Box B$ at some point x in our model is not generally enough to guarantee that both A and B are true at a single point y in any n-tuple related to x. However, by suitably finessing the definition of the modal formula corresponding to a given QBF, we are able to generalize the result, proving:

Theorem 4.1. *For any modal logic S, and any $n \geq 1$, if $K_n \leq S \leq S_4$, then the satisfiability problem for S (S-SAT) is PSPACE-complete.*

While the proof of this claim is outside the scope of this paper (see (Allen, 2003)), we present a generalization of Halpern, Theorem 4.1 (Halpern, 1995), giving an NP-completeness result for K_n-SAT in the presence of bounded operator depth.

The depth of modal formula B, written $dpt(B)$, is the maximum nesting of its modal operators, defined inductively. For any propositional letter p, $dpt(p) = 0$; $dpt(\neg A) = dpt(A)$; $dpt(A \wedge B) = \max(dpt(A), dpt(B))$; and $dpt(\Box A) = (1 + dpt(A))$. This allows a theorem:

Theorem 4.2. *For any $k \in \mathbb{N}$, and any logic K_n, the K_n-satisfiability problem, restricted to $\{A \mid dpt(A) < k\}$, is NP-complete.*

Proof. The reduction from an NP-complete problem is immediate, since every propositional formula A is also a formula of our modal language. Furthermore, $dpt(A) = 0$ for any such A, and thus we

[16]For a simpler presentation of important elements of the proof the reader may also want to see Halpern & Moses (Halpern and Moses, 1992), and Halpern (Halpern, 1995).

[17]Logic K is our K_1, while T adds axiom $(\Box A \rightarrow A)$ to K, and $S4$ adds $(\Box A \rightarrow \Box\Box A)$.

have that propositional SAT reduces immediately to K_n-SAT with depth bounded by any $k \in \mathbb{N}$ and any K_n. We need only show that an NP decision procedure exists for bounded-depth K_n-SAT.

Let B be a modal formula of length $|B| = m$, with depth less than $k \in \mathbb{N}$.[18] Let $\textsc{Prop}(B)$ be the set of propositional letters in B. Now, B is K_n-satisfiable iff some $(n+1)$-ary relational model $\mathfrak{M}$ makes B true. It is easy to see that any such $\mathfrak{M}$ corresponds to a series of assignments, $V_i : \textsc{Prop}(B) \to \{1, 0\}$, one for each point in $\mathfrak{M}$. Once such a series is given, evaluating B is straightforward. A nondeterministic machine N needs only to guess some set of assignments, tracking the right relationships between them, and then perform evaluation as usual. To make the presentation clear, we *label* assignments (although N itself need not use any such device). For logic K_n, each label is some string from the alphabet $\{0, 1, \ldots, n-1, *\}$, of the form:

$$w = \{0, 1, \ldots, n-1\} \circ (* \{0, 1, \ldots, n-1\})^r \quad (r \geq 0).$$

That is, each label is a sequence of numbers from 0 to $(n-1)$, each separated by $*$. Intuitively then, for any w, and any $0 \leq x \leq n-1$, assignment V_{w*x} is a *successor of* assignment V_w. The satisfaction conditions are as usual for propositional letters and boolean connectives; for modal operators, the condition is given by:

$$\forall A,\, V_w \models \Box A \quad \Leftrightarrow \quad (\exists i. 0 \leq i \leq n-1)\; V_{w*i} \models A.$$

(It is easy to see that this condition corresponds to the definition of $V(\Box A)$ [1.1].) So, for any modal subformula B' of B, occurring with $dpt(B') \leq dpt(B) < k$, machine N will need to look at not more than some constant number ($n^{dpt(B')} < n^k$) of assignments in order to determine whether B' is satisfied or not.

To determine the upper limit on the number of assignments V_w our machine N will have to guess for logic K_n and formula B, we observe that for every depth $j \leq dpt(B) < k$, we require at most n such new assignments, one for each modal operator occurring at depth j.[19] Since there must be not more than $|B| = m$ such operators occurring in the entire formula, we require not more than $(n \cdot m)^{dpt(B)} + 1$ such assignments, each of which assigns not more than $|B| = m$ values to propositional letters. The overall size of the assignment-set that must be guessed is thus not more than $(n^{dpt(B)} \cdot m^{dpt(B)+1}) + 1$, which, since $dpt(B) < k$, is strictly less than $(n^k \cdot m^{k+1}) + 1$. Thus, N needs only guess a series of assignments with overall size polynomial in $m = |B|$. $\square$

5 Conclusion: Open problems

The weakly-aggregative modal logics open up a number of avenues for further investigation from a complexity-theoretic point of view, particularly given the capacity of such logics to express interesting graph-theoretic and combinatorial properties. We mention but one here. In the context of a study of modal epistemic logics, Vardi uses a version of neighbourhood semantics in order to address concerns about 'omniscient agents' (Vardi, 1989). In this context, he extends the PSPACE-completeness results to modal logics outside of the range between K and $S4$, and conjectures that this bound depends upon the presence of the fully-aggregative $[K_1]$ axiom: $(\Box A \wedge \Box B) \to \Box(A \wedge B)$. That is, he conjectures, S-SAT is PSPACE-complete iff modal logic S contains said axiom, and is NP-complete otherwise. We have shown that SAT for K_n logics may yet be PSPACE-complete in the presence of axioms strictly weaker than that given, at least in the context of the usual Kripke-style semantics. It is worth investigating how these results translate over into the semantic framework of Vardi, and how they bear on his conjecture.[†]

[18] We measure length $|B|$ in number of symbols. If what we want rather is to measure $|B|$ as some function, say, of $\log|\tau|$, for τ an alphabet of symbols, it is easy to translate what follows into those terms.

[19] Generally less, although the presence of the $\Diamond$ operator, dual to $\Box$, can force us to guess n new assignments each time it appears.

[†] The authours wish to thank the anonymous reviewer for NASSLLI who commented on this paper, and whose suggestions helped improve the presentation of ideas.

References

Allen, M. 2003. On the complexity of modal logics $K_n (n \geq 2)$. Unpublished manuscript.

Apostoli, P. and B. Brown. 1995. A solution to the competeness problem for weakly aggregative modal logic. *Journal of Symbolic Logic*, 60:832–842.

Appel, K. and W. Haken. 1977. Every planar map is four colorable. Part I. Discharging. *Illinois J. Math.*, 21:429–490.

Appel, K., W. Haken, and J. Koch. 1977. Every planar map is four colorable. Part II. Reducibility. *Illinois J. Math.*, 21:491–567.

Bollobás, B. 1991. *Modern Graph Theory.* Springer, New York.

Chellas, B.F. 1980. *Modal logic: an introduction.* Cambridge.

Halpern, Joseph Y. 1995. The effect of bounding the number of primitive propositions and the depth of nesting on the complexity of modal logic. *Artificial Intelligence*, 75:361–72.

Halpern, Joseph Y. and Yoram Moses. 1992. A guide to completeness and complexity for modal logics of knowledge and belief. *Artificial Intelligence*, 54:319–379.

Harary, F. and W. T. Tutte. 1965. A dual form of Kuratowski's theorem. *Canadian Math. Bull.*, 8:17–20, 373.

Jennings, R.E. and P.K. Schotch. 1980. Inference and necessity. *Journal of Philosophical Logic*, IX:327–340.

Jennings, R.E. and P.K. Schotch. 1981. Some remarks on (weakly) weak modal logics. *Notre Dame Journal of Formal Logic*, 22:309–314.

Jennings, R.E., P.K. Schotch, and D.K. Johnston. 1980. Universal first order definability in modal logic. *Zeitschrift für Mathematische Logik und Grundlagen der Mathematik*, 26:327–330.

Jennings, R.E., P.K. Schotch, and D.K. Johnston. 1981. The n-adic undefinability of the Geach formula. *Notre Dame Journal of Formal Logic.*

Ladner, Richard E. 1977. The computational complexity of provability in systems of modal propositional logic. *SIAM J. Computing*, 6:467–480.

Nicholson, T. 2001. A weakening of chromatic number. In K. Striegnitz, editor, *Proceedings for the Student Session of the 13th European Summer School in Logic, Language, and Information*, pages 215–226, University of Helsinki, Finland, August.

Nicholson, T., R.E. Jennings, and D. Sarenac. 2000. Revisiting completeness for the K_n modal logics: a new proof. *the Logic Journal of the IGPL*, 8(1):101–105.

Nicholson, T., R.E. Jennings, and D. Sarenac. 2001. $(n+1)$-Harmonic hypergraphs. In *Institute for Logic, Language and Computation, University of Amsterdam, Netherlands*, August. Invited paper.

Roberston, N., D.P. Sanders, P.D. Seymour, and R. Thomas. 1996. A new proof of the four colour theorem. *Electronic Research Announcements of the American Mathematical Society*, 2(1):17–25.

Saaty, T. L. 1972. Thirteen colorful variations on Guthrie's four-color conjecture. *Am. Math. Monthly*, 79:2–43.

Schotch, P. and R.E. Jennings. 1989. On detonating. *Philosophia, Paraconsistent Logic*, pages 306–327.

Schotch, P.K. and R.E. Jennings. 1980. Modal logic and the theory of modal aggregation. *Philosophia*, 9:265–78.

Schotch, P.K., R.E. Jennings, and D.K. Johnston. The general theory of first-order relational frames. Circulated in mimeograph.

Stockmeyer, L.J. and A.R. Meyer. 1973. Word problems requiring exponential time: preliminary report. In *Proc. Fifth Annual ACM Symp. on Theory of Computing*, pages 1–9, May.

Thomas, R. 1998. An update on the four colour theorem. *Notices Amer. Math. Soc.*, 45:848–859.

Urquhart, A. 1995. Completeness for weakly aggregative modal logic. Unpublished manuscript.

Vardi, Moshe Y. 1989. On the complexity of epistemic reasoning. In *Proc. 4th IEEE Symp. on Logic in Computer Science*, pages 243–252, June.

Wagner, K. 1937. Über eine eigenschaft der ebenen komplexe. *Math. Ann.*, 114:570–90.

Wong, P. 2003. Reasoning and modelling: Two views of inconsistency handling. Unpublished, March.

Biscuit Conditionals and Common Ground

Eric Swanson
swanson@mit.edu

Massachusetts Institute of Technology
Department of Linguistics and Philosophy

Abstract

I argue that biscuit conditionals are best characterized in terms of a sequence of operations on the common ground. First, a biscuit conditional's antecedent is added to the common ground as a supposition to make an otherwise inappropriate assertion appropriate. Second, its consequent makes that assertion. Third, the supposition is lifted. I argue that my account handles some neglected data better than another recent approach defended by DeRose and Grandy.[†]

1 Introduction

Although the philosophical literature focuses on indicatives and subjunctives, there are remarkably many different kinds of conditionals. Here are examples of three:

Indicative conditionals: "If Oswald didn't shoot Kennedy, someone else did."

Subjunctive conditionals: "If Oswald hadn't shot Kennedy, someone else would have."

Biscuit conditionals: "If you're hungry, there's pizza in the fridge."

This paper develops what I will call a 'suppositional' account of biscuit conditionals. In brief, the account is as follows. To utter a biscuit conditional of the form 'If A, B' is not to perform a single speech act. Rather, it is to perform two speech acts, one after the other. A natural language analogue of the first is the imperative 'Suppose that A,' and a natural language analogue of the second is the assertion of B. So an utterance of a biscuit conditional 'If A, B' amounts to an assertion of B under the supposition that A.[1] Acts of

[†]Thanks to Danny Fox, Ned Hall and Bob Stalnaker for helpful discussion of the ideas presented in this paper, and to two anonymous referees for their comments.

[1]In this paper I assume that the objects of supposition are propositions, and that propositions are exactly as finely grained as functions from possible worlds into truth values. When a proposition is supposed in a conversational context, it "becomes (temporarily) a part of the background of common assumptions in subsequent conversation" (Stalnaker 1974, 60). (This is a necessary condition for supposition; I won't try to give sufficient conditions for it here.) What makes assertion under a supposition different from assertion simpliciter is that what is asserted (in an act of asserting under a supposition) is asserted in a context in which some proposition has been supposed. For simplicity, in this paper I assume that the common ground can be modelled using no more than Stalnakerian context sets, i.e., using no more than sets of possible worlds that are compatible with the information in the common ground.

I also ignore the fact that in normal circumstances, when a speaker asserts that p, more than just the proposition that p is added to the common ground. For example, the proposition that the speaker asserted that p is also added to the common ground. Throughout, I am abstracting away from this and similar details, as though facts about the act of assertion per se were never added to the common ground.

supposition are by definition temporary changes to the common ground, and so we also need to say what happens to the common ground when the supposition is 'lifted.' Because the issues involved here are fairly complex, however, I will not discuss them in detail until I have done more to motivate the account and to explain its details.

My approach is opposed to that developed by Keith DeRose and Richard Grandy in their recent article on biscuit conditionals (1999). Drawing on Dorothy Edgington's account of indicative conditionals (1995), DeRose and Grandy argue that to utter a biscuit conditional is to make a "conditional assertion." After presenting some data that I think any theory of biscuit conditionals must account for, I will criticize the conditional assertion approach on the grounds that it cannot explain all the pertinent facts. I will then explain my own theory of biscuit conditionals in more detail and show how it can account for those facts.

Biscuit conditionals deserve careful study for a number of reasons. Here are two especially significant ones. First, I think it is important to give a relatively unified account of the semantics of biscuit and indicative conditionals.[2] If I am right about this, then we can use data about biscuit conditionals to help us evaluate candidate theories of other sorts of conditional expressions.[3] Second, if my account is correct, biscuit conditionals are an interesting example of a construction that speakers use specifically to operate on the common ground when they realize that it would not be appropriate to simply assert the proposition they want to assert in the standing common ground. Indeed, it is important to see that there *are* such constructions, because further work on them may help shed light on the nature of common ground and the ways in which it can be updated.

2 What are biscuit conditionals?

Biscuit conditionals get their name from J. L. Austin's 1956 paper "Ifs and Cans," where Austin gives the sentence

(1) There are biscuits on the sideboard if you want them. (158, 160)

Broadly speaking, biscuit conditionals raise two questions. First, in general a speaker has the right kind of evidence to make her qualified to utter (1) just in case she has the right kind of evidence to make her qualified to utter (2).[4]

(2) There are biscuits on the sideboard.

But if she is qualified to utter (2), why does she instead use the (prima facie) weaker sentence (1)? Second, a speaker who utters (1) seems to be committed to the truth of (2) whether or not her addressee wants biscuits. With other kinds of conditionals, by contrast,

[2] Here are three reasons to prefer a unified account. First, there are biscuit conditionals in many languages other than English, and they have similar surface structure to indicatives, just as they do in English. Second, biscuit conditionals and indicative conditionals exhibit much if not all the same presupposition projection behavior. Third, the antecedents of both biscuit and indicative conditionals license negative polarity items (NPIs), such as non-free choice 'any,' 'ever,' 'a damn,' and so on. Biscuit conditionals of the form "If you want any, there are some . . . " are clear examples of this phenomenon.

Of course, these facts do not show that we *must* give biscuit and indicatives conditionals a uniform semantics. It is open to the objector to insist that we need different semantics for these two kinds of conditionals, and thus to maintain that nothing about the semantics of indicatives turns on the problems that the conditional assertion account, or any other account, has with biscuits. I will not try to show that this objection is wrong, but I think that the similarities I have already mentioned are very suggestive.

[3] I do not have time to say more about indicative conditionals in this paper, but in unpublished work I have been developing an account on which indicatives and biscuits have a relatively unified semantics.

[4] When I say here that a speaker is 'qualified' to utter some sentence ψ, I mean that her evidence for the proposition expressed by ψ meets the standards that must be met for it to be appropriate for her to utter ψ. In this sense, speakers are often qualified to say things that it would be conversationally inappropriate for them to say, although they usually will have evidence that it would be inappropriate to say such things.

it seems that the speaker's commitment to the truth of the consequent is (or, at least, can be) conditional on the truth of the antecedent. I think Austin's description of the way biscuit conditionals involve non-conditional commitment to the truth of their consequents is apt: "I do not know whether you want biscuits or not, but in case you do, I point out that there are some on the sideboard" (160). But consider other kinds of conditionals, like

(3) a. If the glass fell, it broke.
 b. If the glass had fallen, it would have broken.

It would clearly be wrong to paraphrase (3a), for example, by saying that 'I do not know whether the glass fell or not, but in case it did, I point out that it broke.'[5]

Here are two more examples of biscuit conditionals, from DeRose and Grandy's article:

(4) If you're interested, there's a good documentary on PBS tonight.

(5) Oswald shot Kennedy, if that's what you're asking me. (405)

These sentences are most naturally used as biscuit conditionals, and I think that Austin's basic observation about (1) applies well to all of them. A speaker who utters (4) doesn't know whether his addressee is interested in watching a documentary on PBS, and a speaker who utters (5) doesn't know whether his addressee is asking about who shot Kennedy.[6]

2.1 Relevance and truth

It is important to recognize, however, that not all biscuit conditionals involve the speaker's being unsure about the truth of the antecedent. Here is an example.

(6) If you get thirsty soon, there is juice on your tray.

The context that I have in mind is this: the speaker is a doctor, the addressee is her patient. The doctor has just given the patient a drug that will make him very thirsty in about half an hour, and the doctor knows this. So the doctor knows that the antecedent of the biscuit conditional obtains. But the patient isn't at all thirsty now—we can imagine, for example, that the patient's lack of thirst is precisely the reason that the doctor gave the drug in the first place—and the doctor knows this, too. So the doctor doesn't believe that the patient knows or even believes that the antecedent of (6) is true.[7] It is the *addressee's* failure to believe the proposition expressed by the antecedent that warrants the speaker's use of the biscuit conditional. This case shows that the doubt indicated by the use of a biscuit conditional need not be doubt on the speaker's part about the truth of its antecedent. Instead, it may be the speaker's doubt that the addressee thinks the antecedent is true. A good theory of biscuit conditionals should account for this sort of use.

2.2 Presupposition projection

Another important phenomenon is presupposition projection. Although there is much work in the linguistics literature on the presuppositions of other kinds of conditionals, there is so

[5]Thanks to Bob Stalnaker for helpful comments on the issues discussed in this paragraph.

[6](5) is a slightly strange example, since it's not entirely clear which question it is that the speaker intends to refer to in the antecedent of the biscuit conditional. That is, he could be unsure whether the addressee wants to know who shot Kennedy, or to know who Oswald shot, or to know how Oswald killed Kennedy, or to know whether Oswald shot Kennedy, or to know some variation on these. Focus or auxiliary insertion (e.g., "Oswald [shot]$_F$ Kennedy, if that's what you're asking me," "Oswald did shoot Kennedy, if that's what you're asking me,") would generally help disambiguate, but DeRose and Grandy do not indicate whether any particles in the sentence are to be read as focused.

[7]Indeed, instead of (6) the doctor might have said: "You'll get thirsty soon. There is juice on your tray." The biscuit conditional is simply a less direct way of informing the patient that there is juice on his tray, one which doesn't involve informing the patient that he will get thirsty soon.

little work that looks specifically at the presuppositions of biscuit conditionals that it will be worthwhile to spend some time on the issue. Consider the following sentences:

(7) a. If it matters to you, the car is over there.
b. If it matters to you, the car isn't over there.
c. If it matters to you, maybe the car is over there.
d. If it matters to you, it's unlikely that the car is over there.

(8) a. If you don't already know, it was Dawn who bought the flowers.
b. If you don't already know, it wasn't Dawn who bought the flowers.
c. If you don't already know, maybe it was Dawn who bought the flowers.
d. If you don't already know, it's unlikely that it was Dawn who bought the flowers.

(9) a. Ken will have lobster again today, if that's what you mean.
b. Ken won't have lobster again today, if that's what you mean.
c. Maybe Ken will have lobster again today, if that's what you mean.
d. It's unlikely that Ken will have lobster again today, if that's what you mean.

(10) a. If you're wondering, Max knows that the game is tonight.
b. If you're wondering, Max doesn't know that the game is tonight.
c. If you're wondering, maybe Max knows that the game is tonight.
d. If you're wondering, it's unlikely that Max knows that the game is tonight.

(11) a. Sue has started swimming, if that's what you're asking me.
b. Sue hasn't started swimming, if that's what you're asking me.
c. It's unlikely that Sue has started swimming, if that's what you're asking me.

(7a)–(7d) presuppose that there is exactly one salient car; (8a)–(8d) presuppose (among other things) that someone bought the flowers; (9a)–(9c) presuppose that Ken has already had lobster once today; (10a)–(10d) presuppose (among other things) that the game is tonight; and (11a)–(11c) presuppose that Sue hasn't been swimming in the recent past.[8] Thus the presuppositions of definites, clefts, iterative adverbs, factives, and change-of-phase predicates can all project out of the consequents of biscuit conditionals.

For my purposes, what is particularly important to note is that this kind of presupposition projection occurs whether or not the antecedent of the biscuit conditional is true. Suppose that the addressee of (9a) *didn't* mean that Ken will have lobster again today: she meant that he will have lobster for dinner, for the first and only time today. She could obviously still complain that the person who uttered (9a) had presupposed that Ken had lobster earlier today, thereby committing himself to the truth of that proposition.

Other tests also show that presuppositions project out of the consequents of biscuit conditionals whether or not their antecedents are true. For example, consider the so-called "Hey, wait a minute!" test applied to some of the sentences I just mentioned. Intuitively, the "Hey, wait a minute!" test gives infelicities when what has just been asserted is echoed by the addressee, and gives felicities when what has just been presupposed is so echoed.

(12) a. If you don't already know, it was Dawn who bought the flowers.
b. Hey, wait a minute! I had no idea someone *bought* the flowers.

[8] There is, perhaps, a reading of (9b) and a reading of (11b) on which the negation particle serves to effect something like metalinguistic negation, and the standard presuppositions arguably do not project on that reading. This phenomenon is not unusual, and it does not count against my claim that presuppositions can and often do project out of the consequents of biscuit conditionals.

c. # Hey, wait a minute! I had no idea that Dawn did that.

(13) a. If you're wondering, Max doesn't know that the game is tonight.

b. Hey, wait a minute! I had no idea the game is tonight.

c. # Hey, wait a minute! I had no idea Max doesn't know that.

Again, this test shows that presuppositions project out of biscuit conditionals' consequents irrespective of the truth of their antecedents. For example, it is easy to imagine for (13a) and (13b) that the addressee wasn't wondering whether Max knows that the game is tonight because the addressee himself didn't know that the game is tonight. That is just the kind of situation that might cause the addressee to respond to (13a) with (13b).[9]

3 Biscuit conditionals and 'conditional assertion'

DeRose and Grandy think that the key differences between indicative and biscuit conditionals lie in their assertibility conditions. On their account, speakers use standard indicative conditionals when they believe that the "simple probability" of the consequent's truth isn't high enough to warrant asserting the consequent by itself, but do believe that the *conditional* probability of the consequent's truth (C) given the truth of the antecedent (A) is high enough to warrant assertion (412–413). A speaker uses a biscuit conditional, by contrast, when she isn't "sufficiently sure that C is conversationally relevant, but does know that C is relevant if A is true" (413). Thus DeRose and Grandy clearly believe that the relevance of the consequent of a biscuit conditional turns on the *truth* of its antecedent. This aspect of their position makes it unclear how they would handle biscuit conditionals like

(6) If you get thirsty soon, there is juice on your tray.

In the relevant context for (6), the doctor knows (and knows that she knows) that the patient will get thirsty soon, so there is no question in her mind as to the truth of the antecedent. But the doctor believes that the patient believes the antecedent is false, and as a result the doctor also believes that the consequent isn't conversationally relevant. The doctor uses a biscuit conditional precisely because she believes that the *patient* doesn't believe the antecedent is true. That is, she uses a biscuit conditional because the antecedent isn't part of the common ground—not because the antecedent isn't true. For other examples of biscuit conditionals, like (1), (4), and (5), it is more plausible to think that the relevance of the consequent turns on the truth of the antecedent, because in those cases it is the speaker who isn't sure whether the addressee in fact wants any biscuits, or in fact is interested in

[9] It is hard to find cases of presupposition filtering with biscuit conditionals for two reasons. First, the antecedents and consequents of biscuit conditionals are typically so distantly related that it is hard to construct a case in which they are closely enough related that the presuppositions of the consequent are satisfied by the antecedent. Second, if the consequent of a biscuit conditional *did* rely on its antecedent for some of its presuppositions to be satisfied, then in contexts in which those presuppositions weren't already satisfied, its consequent would not be assertible (without the supposition or assertion of its antecedent, or something close to it). But one of the hallmarks of biscuit conditionals is that they are used when an otherwise assertible sentence threatens to be *irrelevant*—not when it threatens to exhibit presupposition failure.

That said, there are sentences that arguably can be interpreted as biscuit conditionals and that arguably exhibit the presupposition filtering behavior associated with indicative conditionals. Here are a few examples:

Additive particles: If you want to go to the movies, Henry and Iris are going, too.

Factive verbs: If you're hungry, there are cookies on the table whether or not you want to admit that you're hungry.

Definite NPs: If anyone wants one, there are puppies looking for their new owners in the backyard.

These sentences raise many subtle issues, but unfortunately I will have to leave them for another time.

PBS documentaries, or in fact is asking who shot Kennedy. But this is just one way for the common ground to be such that the consequent isn't relevant until the antecedent of the biscuit conditional is uttered. The doctor / patient situation I discussed as a context for (6) is another way for the common ground to be such that the consequent isn't relevant until the use of a biscuit conditional makes it so, and in this case the failure of the consequent to be relevant has nothing to do with the truth or falsity of the biscuit conditional's antecedent.

On the other hand, DeRose and Grandy hold that the key similarity between indicative and biscuit conditionals is that they are used to make conditional assertions.

> . . . whenever stating '$\mathsf{A} \to \mathsf{C}$'[10] does result in a truth-evaluable assertion, that assertion is true iff C is true. When does '$\mathsf{A} \to \mathsf{C}$' result in an assertion of C? When A is true. What, then, is asserted where A is false? Nothing. '$\mathsf{A} \to \mathsf{C}$' is used to *conditionally assert* that C. (407, cf. 411)

In other words, what is asserted by an utterance of 'If A, C' depends on whether A is true. If A is true, an utterance of 'If A, C' will be an assertion of the proposition expressed by C. If A is false, an utterance of 'If A, C' will not be an assertion of any proposition whatsoever. If as a matter of fact you're not hungry, when I say to you 'There are biscuits on the sideboard if you're hungry,' I have not succeeded in asserting any proposition. What I have done, according to DeRose and Grandy, is to *implicate* that there are biscuits on the sideboard.[11] Thus DeRose and Grandy rightly acknowledge that a speaker has committed herself to *something* even if the antecedent of the biscuit conditional she utters is false. I think they are wrong, however, to think that that commitment is effected through an implicature. I say this because if the antecedent of the biscuit conditional is false, then according to their own analysis no assertion has been made. It is hard to see how a speaker could implicate anything without having asserted something. Gricean conversational implicature is standardly thought of as a process that relies on the speaker's "saying (or making as if to say) that p" (Grice 1987, 30). But on DeRose and Grandy's view, when a speaker utters a biscuit conditional, she does not say anything unless the antecedent of the biscuit conditional is true. The burden is on DeRose and Grandy to explain how the calculation of the implicature works if the speaker has not said or made as if to say *anything*.[12]

DeRose and Grandy's claim that biscuit conditionals with false antecedents express what they do through implicature also makes it hard for them to explain presupposition projection out of biscuit conditionals. Perhaps DeRose and Grandy would say that presuppositions can somehow project out of implicatures. After all, presuppositions can project from certain non-assertoric contexts, since they generally project out of questions and commands. But it seems almost like a category mistake to think that *implicatures* have presuppositions. Indeed, it seems to be nearly as bad as saying that implicatures themselves have implicatures. It would certainly be interesting if DeRose and Grandy could offer independent evidence of presupposition projection out of implicatures, but without such evidence it is hard to justify positing such projection simply to save their account of biscuit conditionals.

[10] DeRose and Grandy use the standard '$\mathsf{A} \to \mathsf{C}$' to stand for sentences of the form 'If A, C.'

[11] As they put it: " . . . even where the proposition on which your assertion of C is conditioned proves false, and you end up not asserting C, you do implicate that C and generate a simple commitment to the truth of C" (414).

[12] Perhaps DeRose and Grandy would say that a speaker "makes as if" he has asserted the consequent of a biscuit conditional when its antecedent is false, thereby implicating that the consequent is true. I think this response is a non-starter, however. For if we in fact only "made as if" to say something whenever we uttered a conditional with a false antecedent, it would in many cases be just as easy to violate the maxim of relevance by making as if to say that p as it is to violate it by *saying* that p.

4 Biscuit conditionals and common ground

I think that the suppositional account I began to sketch earlier fares better with the phenomena I have been discussing. My account relies on three key ideas to handle biscuit conditionals. The first has to do with the function of antecedents; the second has to do with the function of consequents; and the third has to do with how the context set is updated when the supposition introduced by the antecedent is lifted.

The first important idea behind my approach is that speakers use biscuit conditionals when they want to assert some proposition the assertion of which might violate the maxim of relevance in a way that cannot be easily accommodated. To avoid violating that maxim, a speaker uses a biscuit conditional in order to get the addressee to temporarily suppose that the antecedent of the biscuit conditional is part of the common ground. Once the antecedent becomes part of the common ground, the assertion of the consequent will no longer threaten to violate the maxim of relevance.

There are a number of ways that the common ground can be such that the assertion of a given proposition would violate the maxim of relevance. One way is for the speaker to be unsure whether the proposition expressed by the antecedent of the biscuit conditional is true. But there are other ways for the context to be such that the utterance of a biscuit conditional is felicitous, as my example (6) is supposed to illustrate.

(6) If you get thirsty soon, there is juice on your tray.

Because the doctor doesn't believe that the patient believes he will get thirsty soon, the proposition that the patient will get thirsty soon is not part of the common ground. My account addresses this use of biscuit conditionals directly, for it says that the antecedent of (6) introduces the proposition that the patient will get thirsty soon into the common ground as a supposition.[13] But my account can also handle more familiar cases like (1), (4) and (5), because there too the propositions that the addressee wants biscuits, that the addressee is interested in PBS documentaries, and that the addressee is asking whether Oswald shot Kennedy are not part of the common ground. Again, the antecedent of the biscuit conditional serves to make them part of the common ground, as temporary suppositions.

The second important idea is that the proposition expressed by the consequent of a biscuit conditional is *always* asserted, whether or not the antecedent is true, in the sense that the consequent is always added to the appropriate stage of the context set. In particular, it is added to the stage between the supposition that the antecedent is true and the lifting of that supposition. Indeed, what makes appropriate the assertion of the consequent is the way that the antecedents of biscuit conditionals temporarily update the common ground. Thus the worries I raised about presupposition projection out of the consequents of biscuit conditionals are easy for my account to handle. The presuppositions of a biscuit conditional's consequent project because the consequent updates the common ground whether or not the biscuit conditional's antecedent is true.

Finally, the third important idea specifies the ways in which biscuit conditionals modify the common ground. Call the state of the context set $\mathcal{C}_n$ ($n \in \mathbb{N}$), where subscripts indicate stages in the updating of the context set. Call the initial state of the context set $\mathcal{C}_0$. Now

[13] Of course, there are other ways for a proposition to fail to be part of the common ground. For example, suppose the patient has been researching his condition, and knows that the drug the doctor gave him will make him thirsty soon. Suppose, moreover, that the doctor knows this—she has noticed him reading certain books, say, and looking carefully at the labels of the drugs he has been administered—but that the patient isn't aware of what the doctor knows, and that the doctor knows this as well. In such a situation the proposition that the patient will be thirsty soon is not part of the common ground, because although all the participants in the conversation believe it, not all of the participants believe that all the participants believe it. For this reason my account predicts, rightly, that in such a situation it will be felicitous for the doctor to use a biscuit conditional like (6). Thanks to Ned Hall for this observation.

suppose that a sentence of the form 'If A, B' is uttered in $\mathcal{C}_0$, and that the utterance is felicitous, the addressee is cooperative, and so on. The first operation that will take place is the (temporary) supposition of A:

Step 1: $\mathcal{C}_1 = \mathcal{C}_0 \cap \mathsf{A}$[14]

Next we have the assertion of B, under the supposition that A:

Step 2: $\mathcal{C}_2 = \mathcal{C}_1 \cap \mathsf{B}$

Then the supposition that A is lifted. Now when a speaker uses a biscuit conditional she is as a rule not worried that she might wrongly assert B in the sense that B might be false. Rather, she is typically sufficiently sure that B is true to simply assert B in an appropriate context, and she is as committed to the truth of B as she is to the truth of any other proposition she would ordinarily assert. Her reluctance to *simply* assert B is due solely to relevance constraints imposed by the context in which she finds herself. She uses a biscuit conditional in order to meet these constraints, and her addressees, as competent language users, understand this. So when an addressee interprets a conditional as a biscuit conditional, he accordingly treats the supposition as a mere relevance-ensuring device. The addressee realizes that the speaker's assertion of B under the supposition of A in Step 2 might well have been an assertion simpliciter of B, had the context set been appropriate for such an assertion. So the lifting of the supposition in Step 3 returns to the context set only those $(\mathsf{B} \wedge \neg\mathsf{A})$-worlds that were part of the initial context set, for those were the only B-worlds disregarded by the assertion of Step 2. Step 3 corrects for the effects of the supposition by returning to the context set those worlds that had been excluded for the sake of B's relevance:

Step 3: $\mathcal{C}_3 = \mathcal{C}_2 \cup \big((\mathsf{B} \cap \mathcal{C}_0) \cap -\mathsf{A}\big)$

This step captures the way in which, when a speaker uses a biscuit conditional, she non-conditionally commits herself to the truth of the consequent. The final effect of these calculations is of course no different from

Net effect: $\mathcal{C}_3 = \mathcal{C}_0 \cap \mathsf{B}$

But as we saw earlier, part of the function of biscuit conditionals is to update the context in a way that will prevent the violations of the maxim of relevance that would occur if B were simply asserted. The context change necessary to avoid such violations is captured by Step 1, above.

5 Conclusion

This paper has presented some new data, having to do with considerations about relevance and with presupposition projection, that any successful account of biscuit conditionals must be able to account for. I argued that one approach defended in the literature cannot explain that data, and presented a new theory, that involves a sequence of operations on the common ground, that can. In future work I plan to extend that theory to other kinds of natural language conditional constructions.

[14]For clarity, I have put my context change calculations in terms of set-theoretic operations on the context set. 'A' and 'B' stand for sets of worlds, the characteristic functions of which are propositions.

References

Austin, J. L. 1956. "Ifs and Cans." In *Philosophical Papers*, edited by J. O. Urmson and G. J. Warnock, 1st ed. Oxford: Oxford University Press, 1961.

DeRose, Keith, and Richard E. Grandy. 1999. "Conditional Assertions and 'Biscuit' Conditionals." *Noûs* 33: 405–420.

Edgington, Dorothy. 1995. "On Conditionals." *Mind* 104: 235–329.

Grice, Paul. 1987. "Logic and Conversation." In *Studies in the Way of Words*. Cambridge: Harvard University Press, 1989.

Stalnaker, Robert C. 1974. "Pragmatic Presuppositions." In *Context and Content*. Oxford: Oxford University Press, 1999.

Another characterization of provably recursive functions

Yevgeniy Makarov
Indiana University
emakarov@cs.indiana.edu

May, 2003

Abstract

A new characterization of provably recursive functions of first-order arithmetic is described. Its main feature is using only terms consisting of 0, the successor s and variables in the quantifier rules, namely, universal elimination and existential introduction.

1 Introduction

This paper presents a new characterization of provably recursive functions of first-order arithmetic. We consider functions defined by sets of equations. The equations can be completely arbitrary, not necessarily defining primitive recursive, or even total, functions. The main result states that a function is provably recursive iff its totality is provable (using natural deduction) from the defining set of equations, with one restriction: only terms consisting of 0, the successor s and variables can be used in the derivation rules dealing with quantifiers, namely universal elimination and existential introduction.

A deduction system with such restrictions can be considered as a way of reasoning about non-denoting terms. A set of equations P can define non-total functions over natural numbers (for a precise definitions see Section 2) and a deduction system with regular quantifier rules has quantified variables ranging over all, not necessarily denoting, terms. For example, a formula $\forall x\, \exists y\, f(x) = y$ is trivially provable in a regular system regardless of the definition of f. In contrast, having the above restriction on quantifiers makes them range over terms denoting natural numbers, so the formula $\forall x\, \exists y\, f(x) = y$ is no longer trivially provable. Moreover, its provability implies that the totality of f is provable in first-order arithmetic, i.e., that f is provably recursive, as the main result of this paper shows. This result may be interpreted as showing that the deductive power of our system is similar to the one of the standard Peano Arithmetic.

Our presentation is heavily influenced by (Leivant, 2002) where another framework for reasoning about non-denoting terms, called intrinsic theory, is defined. Intrinsic theory of natural numbers has a unary predicate symbol N which is supposed to mean that its argument is (equal to) a natural number. Intrinsic theory has no restriction on the quantifier rules. In effect, quantifiers in our system are equivalent to quantifiers relativized to N in intrinsic theory. This fact is used to prove one direction of our main result. The other direction is also proved following the reasoning of a similar statement in (Leivant, 2002), but without a reference to intrinsic theory.

A combination of an intrinsic theory (for binary words) and restricted quantifier rules was studied by in (Marion, 2001). The focus there is on characterization of the complexity classes of functions. The main result states that the class of functions provably total in the intrinsic theory with the restriction on quantifiers and using only conjunctions of predicates for induction formulas coincides with the class of polynomial time computable functions. The presence of both the predicate W (an analog of N for words) and the restriction on the quantifier rules plus restricted rules for equality seem to make instantiation of universally quantified variables more limited. For example, even if $W(t)$ is derived, the term t cannot be used in universal elimination unless t is build of constructors only.

$$\frac{A \quad B}{A \wedge B}\,(\wedge I) \qquad \frac{A \wedge B}{A}\,(\wedge E) \qquad \frac{A \wedge B}{B}\,(\wedge E)$$

$$\begin{array}{c} A \\ \vdots \\ B \\ \hline A \to B \end{array}\,(\to I) \qquad \frac{A \to B \quad A}{B}\,(\to E)$$

$$\frac{A}{A \vee B}\,(\vee I) \qquad \frac{B}{A \vee B}\,(\vee I) \qquad \begin{array}{ccc} & A & B \\ & \vdots & \vdots \\ A \vee B & C & C \\ \hline & C & \end{array}\,(\vee E)$$

$$\frac{A[y]}{\forall x\, A[x]}\,(\forall I) \qquad \frac{\forall x\, A[x]}{A[t]}\,(\forall E)$$

y is not free in open assumptions $\quad$ t is free for x in A

$$\frac{A[t]}{\exists x\, A[x]}\,(\exists I) \qquad \begin{array}{cc} & A[y] \\ & \vdots \\ \exists x\, A[x] & C \\ \hline & C \end{array}\,(\exists E)$$

t is free for x in A $\quad$ y is not free in C

Figure 1

2 Definitions

Let P be a set of first-order equations. Let $\mathcal{L}$ be the language of P plus a constant $\mathsf{0}$ and a unary functional symbol s (if they are not already used in P). A theory $\mathbf{A}[P]$ is a first-order theory with equality in the language $\mathcal{L}$. The axioms of $\mathbf{A}[P]$ are the universal closures of the equations in P, the separation axioms $\forall x\, \mathsf{s}(x) \neq \mathsf{0}$ and $\forall x, y\, \mathsf{s}(x) = \mathsf{s}(y) \to x = y$, and induction

$$A[\mathsf{0}] \to \forall x\, (A[x] \to A[\mathsf{s}(x)]) \to \forall x\, A[x]$$

for all formulas A in $\mathcal{L}$. The derivation rules are the usual rules of classical natural deduction shown in Figure 1 plus the rules of equality:

$$\frac{A[t] \quad t = s}{A[s]} \qquad \frac{}{t = t}$$

for all formulas A and terms t, s in $\mathcal{L}$ (here $A[s]$ is obtained from $A[t]$ by replacing *some* occurrences of t by s). It is easy to see that the rules of equality make it a congruence. For example, let AM be the usual axioms for addition and multiplication and let PR be the set of standard defining equations for all primitive recursive functions. Then $\mathbf{A}[\mathrm{AM}]$ is Peano Arithmetic and $\mathbf{A}[\mathrm{PR}]$ is Peano Arithmetic with primitive recursive functional symbols.

A *program* is a pair (P, f) consisting of a set of equations P and a functional symbol f occurring in P. (When the functional symbol f is clear from the context or when it is irrelevant we will write P instead of (P, f).)

We use programs to define functions using an analog of Herbrand-Gödel computability (see, for example, (Kleene, 1952)). Given a program P, we write $P \vdash^{=} E$ if E is an equation derivable from P in equational logic. The rules of equational logic are the following:

1. $P \vdash^{=} E$ for every $E \in P$;
2. $P \vdash^{=} t = t$ for every term t;
3. if $P \vdash^{=} E[x]$ then $P \vdash^{=} E[t]$ for every term t and a variable x;
4. if $P \vdash^{=} s[t] = r[t]$ and $P \vdash^{=} t = t'$ then $P \vdash^{=} s[t'] = r[t']$.

The relation *computed by* (P, f) is $\{(\vec{n}, m) \mid P \vDash \mathsf{f}(\vec{\bar{n}}) = \bar{m}\}$ (as usual, $\bar{n}$ is a numeral for a number n, consisting of n occurrences of s applied to $\mathsf{0}$). This relation does not have to be a function. Let us call P *coherent* if $P \not\vDash \bar{m} = \bar{n}$ for two distinct numerals $\bar{m}$ and $\bar{n}$. It is easy to see that the relation computed by a coherent program is a partial function.

However, even for a coherent program P the theory $\mathbf{A}[P]$ can be inconsistent because of the separation axioms. This is the case, for example, for $P = \{\mathsf{f}(\mathsf{g}(\mathsf{0})) = \mathsf{s}(\mathsf{g}(\mathsf{0})), \mathsf{f}(x) = \mathsf{g}(\mathsf{0})\}$ with new functional symbols f and g. Call a program P *strongly coherent* if $\mathbf{A}[P]$ is consistent. It is clear that if a program is strongly coherent then it is coherent.

In the future it will be important that a program containing a functional symbol f corresponding to a primitive recursive function f in PR also contains all defining equations for f. Programs that satisfy this property are called *full*.

A term t is called *function-free* if t consists of $\mathsf{0}$, s and variables only. A term t is called *primitive recursive* if t is in the language of PR. If T is a theory then a formula A is called *provable with function-free terms* (respectively, *provable with primitive recursive terms*) in T if there is a classical natural deduction derivation of A from T where the eigenterms of the rules of universal elimination and existential introduction (i.e., terms t in the rules $(\forall E)$ and $(\exists I)$ in Figure 1) are function-free (respectively, primitive recursive). Formulas provable with function-free (primitive recursive) terms in T are also called ff-provable (pr-provable) in T, and this is denoted $T \vdash^{\mathrm{ff}} A$ ($T \vdash^{\mathrm{pr}} A$). More generally, if there is a natural deduction derivation in T of a formula A from assumptions Γ with the above restrictions on quantifiers, this is denoted by $T \vdash^{\mathrm{ff}} \Gamma \Rightarrow A$ or $T \vdash^{\mathrm{pr}} \Gamma \Rightarrow A$.

A function f is called ff-provable if f is computed by a strongly coherent full program (P, f) and $\mathbf{A}[P] \vdash^{\mathrm{ff}} \forall \vec{x}\, \exists y\, \mathsf{f}(\vec{x}) = y$, and similarly for pr-provable.

3 Provably recursive functions are ff-provable

We choose the following definition of provably recursive functions of a theory T: f is called provably recursive if $f(\vec{x}) = h(\mu y . g(\vec{x}, y) = 0)$ where g and h are primitive recursive and $T \vdash \forall \vec{x}\, \exists y\, \mathsf{g}(\vec{x}, y) = \mathsf{0}$.

The proof of the claim that every provably recursive function is ff-provable uses several lemmas.

Lemma 1 Suppose f is a primitive recursive function and f is the corresponding functional symbol from PR. Then $\mathbf{A}[\mathrm{PR}] \vdash^{\mathrm{ff}} \forall \vec{x}\, \exists y\, \mathsf{f}(\vec{x}) = y$.

Proof By induction on the construction of f. If f is one of the base functions, i.e., zero, addition of one or projection then the claim is obvious. (Note that here we use the fact that s can appear in the quantifier rules' eigenterms.) Suppose f is defined by composition using the equation

$$\mathsf{f}(\vec{x}) = \mathsf{h}(\mathsf{g}_1(\vec{x}), \ldots, \mathsf{g}_k(\vec{x}))$$

and suppose that the formulas $\forall \vec{x}\, \exists y_i\, \mathsf{g}_i(\vec{x}) = y_i$ $(i = 1, \ldots, k)$ and $\forall \vec{y}\, \exists u\, \mathsf{h}(\vec{y}) = u$ are ff-provable in $\mathbf{A}[\mathrm{PR}]$. For $k = 1$ a derivation for f is shown in Figure 2(a).

Suppose f is defined by primitive recurrence using equations

$$\mathsf{f}(\vec{x}, \mathsf{0}) = \mathsf{g}(\vec{x})$$
$$\mathsf{f}(\vec{x}, \mathsf{s}y) = \mathsf{h}(\vec{x}, y, \mathsf{f}(\vec{x}, y))$$

and suppose the formulas $\forall \vec{x}\, \exists u\, \mathsf{g}(\vec{x}) = u$ and $\forall \vec{x}, y, p\, \exists v\, \mathsf{h}(\vec{x}, y, p) = v$ are ff-provable in $\mathbf{A}[\mathrm{PR}]$. The formula $\forall y\, \exists z\, \mathsf{f}(\vec{x}, y) = z$ is proved using induction on y. The base case is

$$\dfrac{\dfrac{\forall \vec{x}\, \exists u\, \mathsf{g}(\vec{x}) = u}{\exists u\, \mathsf{g}(\vec{x}) = u}\,(\forall E) \qquad \dfrac{\dfrac{\dfrac{\forall \vec{x}\, \mathsf{f}(\vec{x}, \mathsf{0}) = \mathsf{g}(\vec{x})}{\mathsf{f}(\vec{x}, \mathsf{0}) = \mathsf{g}(\vec{x})}\,(\forall E) \qquad \overset{(1)}{\mathsf{g}(\vec{x}) = u}}{\mathsf{f}(\vec{x}, \mathsf{0}) = u}}{\exists z\, \mathsf{f}(\vec{x}, \mathsf{0}) = z}\,(\exists I)}{\exists z\, \mathsf{f}(\vec{x}, \mathsf{0}) = z}\,(1)$$

and the induction step is shown in Figure 2(b). □

Lemma 2 For every primitive recursive term $t[\vec{x}]$, $\mathbf{A}[\mathrm{PR}] \vdash^{\mathrm{ff}} \forall \vec{x}\, \exists y\, t[\vec{x}] = y$.

$$\dfrac{\dfrac{\dfrac{\forall\vec{x}\,\exists y\,\mathsf{g}_1(\vec{x})=y}{\exists y\,\mathsf{g}_1(\vec{x})=y}\,(\forall E)\qquad \dfrac{\dfrac{\forall y\,\exists u\,\mathsf{h}(y)=u}{\exists u\,\mathsf{h}(y)=u}\,(\forall E)\qquad \dfrac{\dfrac{\dfrac{\dfrac{\dfrac{\forall\vec{x}\,\mathsf{f}(\vec{x})=\mathsf{h}(\mathsf{g}_1(\vec{x}))}{\mathsf{f}(\vec{x})=\mathsf{h}(\mathsf{g}_1(\vec{x}))}\,(\forall E)\qquad \overset{(1)}{\mathsf{g}_1(\vec{x})=y}}{\mathsf{f}(\vec{x})=\mathsf{h}(y)}\qquad \overset{(2)}{\mathsf{h}(y)=u}}{\mathsf{f}(\vec{x})=u}}{\exists z\,\mathsf{f}(\vec{x})=z}\,(\exists I)}{\exists z\,\mathsf{f}(\vec{x})=z}\,(2)}{\exists z\,\mathsf{f}(\vec{x})=z}\,(1)}{\forall\vec{x}\,\exists z\,\mathsf{f}(\vec{x})=z}$$

Figure 2(a)

$$\dfrac{\dfrac{\dfrac{\overset{(1)}{\exists z\,\mathsf{f}(\vec{x},y)=z}\qquad \dfrac{\dfrac{\forall\vec{x},y,p\,\exists v\,\mathsf{h}(\vec{x},y,p)=v}{\exists v\,\mathsf{h}(\vec{x},y,p)=v}\,(\forall E)\qquad \dfrac{\dfrac{\dfrac{\dfrac{\dfrac{\forall\vec{x},y\,\mathsf{f}(\vec{x},\mathsf{s}y)=\mathsf{h}(\vec{x},y,\mathsf{f}(\vec{x},y))}{\mathsf{f}(\vec{x},\mathsf{s}y)=\mathsf{h}(\vec{x},y,\mathsf{f}(\vec{x},y))}\,(\forall E)\qquad \overset{(2)}{\mathsf{f}(\vec{x},y)=p}}{\mathsf{f}(\vec{x},\mathsf{s}y)=\mathsf{h}(\vec{x},y,p)}\qquad \overset{(3)}{\mathsf{h}(\vec{x},y,p)=v}}{\mathsf{f}(\vec{x},\mathsf{s}y)=v}}{\exists z\,\mathsf{f}(\vec{x},\mathsf{s}y)=z}\,(\exists I)}{\exists z\,\mathsf{f}(\vec{x},\mathsf{s}y)=z}\,(3)}{\exists z\,\mathsf{f}(\vec{x},\mathsf{s}y)=z}\,(2)}{\exists z\,\mathsf{f}(\vec{x},y)=z\to\exists z\,\mathsf{f}(\vec{x},\mathsf{s}y)=z}\,(1)}{\forall y\,(\exists z\,\mathsf{f}(\vec{x},y)=z\to\exists z\,\mathsf{f}(\vec{x},\mathsf{s}y)=z)}$$

Figure 2(b)

$$\dfrac{\dfrac{\dfrac{\forall\vec{x}\,\exists y\,\mathsf{g}'(\vec{x},y)=\mathsf{0}}{\exists y\,\mathsf{g}'(\vec{x},y)=\mathsf{0}}\,(\forall E)\qquad \dfrac{\dfrac{\forall y\,\exists u\,\mathsf{h}(y)=u}{\exists u\,\mathsf{h}(y)=u}\,(\forall E)\qquad \dfrac{\dfrac{\dfrac{\dfrac{\dfrac{\dfrac{\forall\vec{x},y\,\mathsf{f}(\vec{x})=\mathsf{h}(\mathsf{k}(\mathsf{g}'(\vec{x},y),\vec{x},y))}{\mathsf{f}(\vec{x})=\mathsf{h}(\mathsf{k}(\mathsf{g}'(\vec{x},y),\vec{x},y))}\,(\forall E)\qquad \overset{(1)}{\mathsf{g}'(\vec{x},y)=\mathsf{0}}}{\mathsf{f}(\vec{x})=\mathsf{h}(\mathsf{k}(\mathsf{0},\vec{x},y))}\qquad \dfrac{\forall\vec{x},y\,\mathsf{k}(\mathsf{0},\vec{x},y)=y}{\mathsf{k}(\mathsf{0},\vec{x},y)=y}\,(\forall E)}{\mathsf{f}(\vec{x})=\mathsf{h}(y)}\qquad \overset{(2)}{\mathsf{h}(y)=u}}{\mathsf{f}(\vec{x})=u}}{\exists z\,\mathsf{f}(\vec{x})=z}\,(\exists I)}{\exists z\,\mathsf{f}(\vec{x})=z}\,(2)}{\exists z\,\mathsf{f}(\vec{x})=z}\,(1)}{\forall\vec{x}\,\exists z\,\mathsf{f}(\vec{x})=z}$$

Figure 3

Proof By induction on t, similarly to the case of composition in Lemma 1. $\square$

Lemma 3 If $\mathbf{A}[\mathrm{PR}] \vdash A$ then $\mathbf{A}[\mathrm{PR}] \vdash^{\mathrm{ff}} A$.

Proof By induction on the derivation. The only non-trivial cases are $(\forall E)$ and $(\exists I)$.

Suppose $A[t[\vec{y}]]$ is derived from $\forall x\, A[x]$. Since $t[\vec{y}]$ is a primitive recursive term, $\mathbf{A}[\mathrm{PR}] \vdash^{\mathrm{ff}} \forall \vec{y}\, \exists z\, t[\vec{y}] = z$ by Lemma 2. Then the following is the derivation of $A[t[\vec{y}]]$:

$$\dfrac{\dfrac{\forall \vec{y}\, \exists z\, t[\vec{y}] = z}{\exists z\, t[\vec{y}] = z}\,(\forall E) \qquad \dfrac{\dfrac{\forall x\, A[x]}{A[z]}\,(\forall E) \quad \dfrac{\overset{(1)}{t[\vec{y}] = z}}{z = t[\vec{y}]}}{A[t[\vec{y}]]}}{A[t[\vec{y}]]}\,(1) \quad .$$

Suppose $\exists x\, A[x]$ is derived from $A[t[\vec{y}]]$. As before, we have $\mathbf{A}[\mathrm{PR}] \vdash^{\mathrm{ff}} \forall \vec{y}\, \exists z\, t[\vec{y}] = z$. Then the following is the derivation of $\exists x\, A[x]$:

$$\dfrac{\dfrac{\forall \vec{y}\, \exists z\, t[\vec{y}] = z}{\exists z\, t[\vec{y}] = z}\,(\forall E) \qquad \dfrac{\dfrac{A[t[\vec{y}]] \quad \overset{(1)}{t[\vec{y}] = z}}{A[z]}}{\exists x\, A[x]}\,(\exists I)}{\exists x\, A[x]}\,(1) \quad .$$

$\square$

Theorem 4 All provably recursive functions of $\mathbf{A}[\mathrm{PR}]$ are ff-provable.

Proof Suppose $f(\vec{x}) = h(\mu y.g(\vec{x}, y) = 0)$ and $\mathbf{A}[\mathrm{PR}] \vdash \forall \vec{x}\, \exists y\, \mathsf{g}(\vec{x}, y) = \mathsf{0}$. We would like to change g so that for each $\vec{x}$ it takes 0 for exactly one y, so we define

$$g'(\vec{x}, y) = g(\vec{x}, y) + \sum_{z<y} \overline{\mathrm{sg}}\, g(\vec{x}, z)$$

where $\overline{\mathrm{sg}}(0) = 1$ and $\overline{\mathrm{sg}}(x) = 0$ for $x \neq 0$. It is straightforward to see that $\mathbf{A}[\mathrm{PR}] \vdash \forall \vec{x}\, \exists! y\, \mathsf{g}'(\vec{x}, y) = \mathsf{0}$ and that $f(\vec{x}) = h(\mu y.g'(\vec{x}, y) = 0)$.

By Lemma 3, $\mathbf{A}[\mathrm{PR}] \vdash^{\mathrm{ff}} \forall \vec{x}\, \exists y\, \mathsf{g}'(\vec{x}, y) = \mathsf{0}$. Also, by Lemma 1, $\mathbf{A}[\mathrm{PR}] \vdash^{\mathrm{ff}} \forall y\, \exists u\, \mathsf{h}(y) = u$. Let P be the minimal full program containing equalities from PR for all primitive recursive functional symbols used in these derivations, plus the following equalities:

$$\begin{aligned} &\mathsf{f}(\vec{x}) = \mathsf{h}(\mathsf{k}(\mathsf{g}'(\vec{x}, y), \vec{x}, y)) \\ &\mathsf{k}(\mathsf{0}, \vec{x}, y) = y. \end{aligned}$$

A derivation of $\forall \vec{x}\, \exists z\, \mathsf{f}(\vec{x}) = z$ in $\mathbf{A}[P]$ is shown in Figure 3.

To prove that P computes f we note that for every $\vec{m}, n$, if $f(\vec{m}) = n$ then P proves $\mathsf{f}(\vec{\bar{m}}) = \bar{n}$ in equational logic, so it is enough to show that P is strongly coherent. But P is true in the standard natural numbers model if f is interpreted by f and k is interpreted by

$$k(z, \vec{x}, y) = \begin{cases} y & \text{if } z = 0, \\ \min\{w \mid h(w) = f(\vec{x})\} & \text{otherwise.} \end{cases}$$

The function k is total because f is total. Since $\mathbf{A}[P]$ with this interpretations of functions is true on natural numbers, $\mathbf{A}[P]$ is consistent. $\square$

4 Functions that are ff-provable are provably recursive

This direction can be proved by interpreting equalities $t = s$ as $\mathbf{A}$[PR]-formulas saying that there exists a derivation of $t = s$ from a program P in equational logic. This interpretation can be extended to all formulas (quantifiers should be interpreted as ranging over numerals) and its soundness w.r.t. $\mathbf{A}$[PR] can be proved. Then if $\mathbf{A}[\mathrm{PR}] \vdash^{\mathrm{ff}} \forall\vec{x}\,\exists y\,\mathsf{f}(\vec{x}) = y$ then $\mathbf{A}$[PR] proves a statement expressing that for every numerals $\vec{\bar{m}}$ there exists a numeral $\bar{n}$ and an equational proof of $\mathsf{f}(\vec{\bar{m}}) = \bar{n}$ from P. This would imply that f is provably recursive.

However, we will prove this statement indirectly, using intrinsic theories introduced by Leivant (Leivant, 2002). An intrinsic theory is a framework for reasoning about inductively generated data. It is similar to inductive predicates described in (Schwichtenberg, 2002).

An intrinsic theory for natural numbers, $\mathbf{IT}(\mathbb{N})$, is a first-order theory with equality whose vocabulary has functional symbols $\mathsf{0}$, s and a unary predicate symbol N. The additional derivation rules are:

$$\frac{}{N(\mathsf{0})} \qquad \frac{N(t)}{N(\mathsf{s}t)} \qquad \frac{N(t) \quad A[\mathsf{0}] \quad \forall x\,(A[x] \to A[\mathsf{s}x])}{A[t]}.$$

The variant of intrinsic theory that we are using, called discrete intrinsic theory and denoted by $\overline{\mathbf{IT}}(\mathbb{N})$ in (Leivant, 2002), also includes the separation axioms.

A function f is called provable in $\overline{\mathbf{IT}}(\mathbb{N})$ if it is computed by a strongly coherent program (P, f) and $\overline{\mathbf{IT}}(\mathbb{N}), P \vdash \forall\vec{x}\,(N(\vec{x}) \to N(\mathsf{f}(\vec{x})))$.

The following theorem is proved in (Leivant, 2002).

Theorem 5 A function is provably recursive in $\mathbf{A}$[PR] iff it is provable in $\overline{\mathbf{IT}}(\mathbb{N})$.

(The proof of the ($\Leftarrow$) direction in (Leivant, 2002) follows the outline in the beginning of this section.) Thus it is enough to show that ff-provable functions are provable in $\overline{\mathbf{IT}}(\mathbb{N})$. However, we can show a stronger result, namely, that pr-provable functions are provable in $\overline{\mathbf{IT}}(\mathbb{N})$.

Let us introduce some notation. If A is a formula then A^N denotes A with all quantifiers relativized to N, i.e., having all subformulas of the form $\forall x\,B$ replaced by $\forall x\,(N(x) \to B)$ and all subformulas of the form $\exists x\,B$ replaced by $\exists x\,(N(x) \land B)$. If Γ is a set of formulas then $\Gamma^N = \{A^N \mid A \in \Gamma\}$. If $\vec{x} = x_1, \dots, x_n$ then $N(\vec{x})$ denotes $N(x_1) \land \dots \land N(x_n)$.

Lemma 6 Let P be a full program and let $t[\vec{x}]$ be a primitive recursive term in the language of P. Then $\overline{\mathbf{IT}}(\mathbb{N}), P \vdash N(\vec{x}) \Rightarrow N(t[\vec{x}])$.

Proof The proof is similar to Lemma 2. For example, if f is a symbol for a function $f(\vec{x}, y)$ defined by primitive recurrence on y then one needs to use induction for the formula $N(y) \land N(\mathsf{f}(\vec{x}, y))$. The fullness of P is necessary to ensure that the induction hypothesis is true of all subterms of t. $\square$

Lemma 7 Let P be a full program. Let also $\Gamma \cup \{A\}$ be a set of formulas in the language of P whose free variables are among $\vec{x}$. If $\mathbf{A}[P] \vdash^{\mathrm{pr}} \Gamma \Rightarrow A$ then $\overline{\mathbf{IT}}(\mathbb{N}), P \vdash N(\vec{x}), \Gamma^N \Rightarrow A^N$.

Proof The proof is by induction on the derivation. If A is an axiom of $\mathbf{A}[P]$ then $\overline{\mathbf{IT}}(\mathbb{N}), P \vdash A$ and $A \vdash A^N$. The only other cases that need attention are those dealing with quantifiers and induction.

If $A[t]$ is derived from $\forall y\,A[y]$ then by induction hypothesis, $\forall y\,(N(y) \to A^N[y])$ is derivable. Since t is a primitive recursive term in the language of P, $N(t)$ is derivable by Lemma 6, so $A^N[t]$ is derivable as well. The case of $(\exists I)$ is similar. The cases of $(\forall I)$ and $(\exists E)$ are also straightforward.

The relativized version of the induction axiom is

$$B^N[\mathsf{0}] \to \forall y\,(N(y) \to B^N[y] \to B^N[\mathsf{s}y]) \to \forall y\,(N(y) \to B^N[y]).$$

It is proved by induction in $\overline{\mathbf{IT}}(\mathbb{N})$ for the formula $N(y) \land B^N[y]$. $\square$

Theorem 8 All pr-provable functions are provably recursive.

Proof Let f be computed by a strongly coherent full program (P, f) and let $\mathbf{A}[P] \vdash^{\mathrm{pr}} \forall \vec{x}\, \exists y\, \mathsf{f}(\vec{x}) = y$. Then by Lemma 7, $\overline{\mathbf{IT}}(\mathbb{N}), P \vdash \forall \vec{x}\, (N(\vec{x}) \rightarrow \exists y\, N(y) \wedge \mathsf{f}(\vec{x}) = y)$. This implies that $\overline{\mathbf{IT}}(\mathbb{N}), P \vdash \forall \vec{x}\, (N(\vec{x}) \rightarrow N(\mathsf{f}(\vec{x})))$, so by Theorem 5, f is provably recursive. $\square$

Acknowledgments

I am grateful to Daniel Leivant and Lev Beklemishev for constructive discussion.

References

Kleene, Stephen. 1952. *Introduction to Metamathematics*. Wolters-Noordhof, Groningen.

Leivant, Daniel. 2002. Intrinsic reasoning about functional programs I: first order theories. *Annals of Pure and Applied Logic*, 114(1–3):117–153, April.

Marion, Jean-Yves. 2001. Actual arithmetic and feasibility. In L.Fribourg, editor, *15th International workshop, Computer Science Logic, CSL'01*, volume 2142 of *Lecture Notes in Computer Science*, pages 115–139. Springer-Verlag.

Schwichtenberg, Helmut. 2002. Program extraction from proofs. Course notes.

Kripke Completeness of First-Order Constructive Logics with Strong Negation

Ichiro HASUO
Department of Mathematical and Computing Sciences, Tokyo Institute of Technology
O-okayama, Meguro, Tokyo 152-8552, JAPAN
hasuo2@is.titech.ac.jp, http://www.is.titech.ac.jp/~hasuo2

Abstract

This paper considers Kripke completeness of quantified Nelson's constructive logic N (intuitionistic logic plus *strong negation* $\sim$) and several variants of it: by omitting $A \to (\sim A \to B)$, adding the axiom of constant domain, adding $(A \to B) \vee (B \to A)$, and adding $\neg\neg(A \vee \sim A)$; the last one we would like to call *the axiom of potential omniscience* and it is interpreted that *we can always verify or falsify a statement, with proper additional information.* The proofs are in a unified way, called *tree-sequent.*

The author is sorry to have left a number of detailed definitions, proofs and illustrative figures in the full paper (Hasuo and Kashima, 2003), due to the limit of the space.

1 Introduction

Strong negation *Strong* (or *constructive*) *negation*, denoted by $\sim$ in this paper, is a negation operator in constructive logics introduced in (Nelson, 1949) and (Markov, 1950). The negation $\neg A$ in intuitionistic logic Int, which will be called *Heyting's negation* here, is reduced to $A \to \bot$, reflecting such an idea of intuitionists as "the compound sentences are not a product of experiment, they arise from reasoning. This concerns also negation: we see that the lemon is yellow, we do not see that it is not blue" (Grzegorczyk, 1964). However, the view of constructivists that "we can not only *verify* a simple proposition such as *This door is locked.* by direct inspection, but also *falsify* it" (Kracht, 1998), that is, taking negative information as primitive as positive one, motivates another choice of negation, *strong negation.*

Nelson's constructive logic N is an extension of Int by the strong negation operator $\sim$, where the word *logic* designates a pair of a formal language and a set of its formulas which are admitted as theorems. $\sim$ is axiomatized in the Hilbert-style system as follows:

$$A \to (\sim A \to B),$$
$$\sim(A \wedge B) \leftrightarrow \sim A \vee \sim B, \quad \sim(A \vee B) \leftrightarrow \sim A \wedge \sim B, \quad \sim(A \to B) \leftrightarrow A \wedge \sim B,$$
$$\sim\sim A \leftrightarrow A, \quad \sim\neg A \leftrightarrow A, \quad \sim\forall x A \leftrightarrow \exists x \sim A, \quad \sim\exists x A \leftrightarrow \forall x \sim A.$$

Axiomatized as above, $\sim$ enjoys many properties which imply that both A (positive information) and $\sim A$ (negative information) are equally primitive. One example is the principle of *constructible falsity*, $\vdash \sim(A \wedge B)$ iff $\vdash \sim A$ or $\vdash \sim B$, which can be regarded as a negative counterpart of the *disjunction property.* Another is found in the Kripke interpretation of N, which we will observe in detail later.

(Wansing, 1993) argues that certain kinds of substructural logics with $\sim$ can be candidates for the logic of *information structure*, due to the character stated above. And in recent years, constructive logics have been paid attention to in the field of logic programming, e.g. (Herre and Pearce, 1992), (Pearce and Wagner, 1990), (Pearce and Wagner, 1991), (Wagner, 1994) and (Akama, 1997).

The axiom of potential omniscience Since N is a conservative extension of Int, it is natural to consider extensions of intermediate logics by $\sim$*. However, it seems that there are less studies

*For example, (Goranko, 1985), (Sendlewski, 1984), (Sendlewski, 1990), and (Kracht, 1998) discuss unquantified cases using algebraic methods.

on extra axioms which are peculiar to logics with $\sim$. We will introduce one such axiom in this paper; $\neg\neg(A \vee \sim A)$, which we would like to call *the axiom of potential omniscience.* Intuitively it is interpreted that *we can verify or falsify any statement, with proper additional information.*

In the Kripke interpretation, the axiom corresponds to the following statement: for every closed formula A and every state of information a, there is a state b which is reachable from a and where A is either verified or falsified. Hence the axiom, especially when combined with the axiom $(A \to B) \vee (B \to A)$ (which means that *the set of information states is linearly ordered*), seems useful to formalize such cases as some kind of games, e.g. cryptography. Consider a game in which Alice (the dealer) knows the answer and Bob (the player) tries to find it; Bob can reach the correct answer if he obtains the information that Alice has.

The axiom may also be considered as one of the weaker versions of *the law of excluded middle*, $A \vee \sim A$[†].

What is shown in this paper This paper is to show Kripke completeness of several variants of N, for the quantified case. The proofs are by the usage of a *tree-sequent* (abbreviated as TS) introduced by Kashima (Kashima, 1999), which is a labelled tree each node of which is associated with a sequent. Tree-sequents can be regarded as a kind of semantic tableaux, and are in the tree-shape since we aim at completeness as to tree-shaped models. Needless to say, the method is also applicable to Kripke completeness of Int and some intermediate predicate logics; the results can be found in (Kashima, 1999).

The logics whose Kripke completeness is proved in this paper are presented in Fig. 1, enclosed in a box[‡]. Completeness of those logics with both O and P remain unproved; we shall see the difficulty therein in the last section.

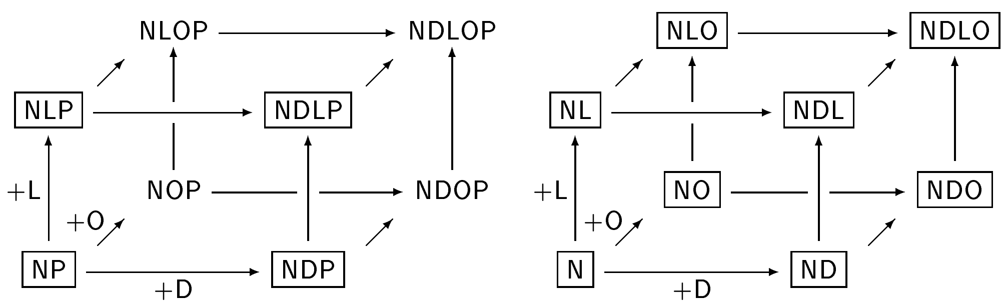


Fig. 1: N-family

Here each of the letters D, L, O and P designates what follows:

- D Adding *the axiom of constant domain*, $\forall x(A(x) \vee B) \to \forall x A(x) \vee B$, where there are no free occurrences of x in B. An intermediate logic CD, which is Int plus this axiom, is characterized by the class of Kripke models whose domain is a constant map (Görnemann, 1971). D is for *domain.*
- L Adding the axiom $(A \to B) \vee (B \to A)$. Dummett's logic LC, which is Int plus this axiom, is characterized by the class of linearly ordered Kripke models (Corsi, 1992)[§]. L is for *linear.*
- O Adding the axiom of potential omniscience, $\neg\neg(A \vee \sim A)$. O is for *omniscience.*
- P Omitting the axiom $A \to (\sim A \to B)$, which is one of those which axiomatize $\sim$ and may be considered as a constructive version of *the ex falso rule* $\perp \to B$. Omitting the axiom results in paraconsistency[¶]. P is for *paraconsistency.*

[†] $\neg A \vee \neg\neg A$ is called *the weak law of excluded middle* and Int plus the axiom is a well-known intermediate logic KC. (Corsi and Ghilardi, 1989) considers Kripke completeness of KC and its extensions, for the quantified case.

[‡] (Almukdad and Nelson, 1984) considers $\neg$-free fragments of ND, NP and NDP (denoted there by N^{+}, N^{-} and N^{+-} respectively) and gives Gentzen-style sequent systems for them.

[§] Kripke completeness of CD or LC can be proved more easily using the TS method. The proof is just the same as that for ND or NL, and presented in (Kashima, 1999).

[¶] (Priest and Routley, 1984) gives an introduction to paraconsistent logics.

The family of sixteen logics presented in the lattices above we would like to call *the* N*-family* in this paper.

For those with O we cannot give proofs just applying the TS method. For them two different kinds of proofs will be presented: one is by an embedding of classical logic Cl into NO, and the other is by the *TSg* method, which is an extension of the TS method.

There are some logics above whose Kripke completeness is already shown; ND is in (Thomason, 1969) using the method of Henkin, and N is in (van Dalen, 1986) reducing N to Int. Nevertheless the author do not take his proofs as useless; the methods used in them are also applicable to other logics.

Notations We often denote logics in the N-family by such a form as N[D][P]; this is for "N, ND, NP and NDP". ND[P] is for "ND and NDP".

We do not consider constants or function symbols, which makes the arguments simpler without essential loss of generality. $\equiv$ denotes syntactical equivalence. $A[y/x]$ is a substitution, obtained by replacing every free occurrence of x in A by y.

As in Tarski-type semantics for Cl, in defining $\models$ and its variants we will introduce temporary constants $\underline{u}$ each of which designates a certain individual u.

For a finite set of formulas $\Gamma = \{A_1, \ldots, A_m\}$, $\bigwedge\Gamma$ (or $\bigvee\Gamma$) is an abbreviation for $A_1 \wedge \ldots \wedge A_m$ (or $A_1 \vee \ldots \vee A_m$). If $\Gamma = \emptyset$, it is $\top$ (or $\bot$), which is an abbreviation for $A \to A$ (or $\neg(A \to A)$, respectively).

2 Syntax and semantics

2.1 Gentzen-style sequent systems GN[D][L][O][P]

We formalize the logics in the N-family by Gentzen-style sequent systems. They share one formal language, consisting of the following symbols: countably many variables $x_1, x_2, \ldots$; countably many m-ary predicate symbols for each $m \in \mathbb{N}$ p_1^m, p_2^m, $\ldots$; and logical connectives $\wedge$, $\neg$, $\to$, $\sim$ and $\forall$. $\vee$ and $\exists$ are introduced as defined symbols: $A \vee B :\equiv {\sim}({\sim}A \wedge {\sim}B)$ and $\exists x A :\equiv {\sim}\forall x {\sim}A$. $A \leftrightarrow B$ is for $(A \to B) \wedge (B \to A)$. Terms and formulas are composed in the same way as those of Cl, and note that $\sim$ is unary.

A *sequent* is defined as an ordered pair of finite sets of formulas separated by the symbol $\Rightarrow$, hence the rule of exchange or contraction can be omitted.

A Gentzen-style sequent system **G**N for N is as follows:

$$\frac{}{A \Rightarrow A}\ (\text{Identity, Id}) \qquad \frac{}{A, {\sim}A \Rightarrow}\ (\text{Ex Falso, Fal})$$

$$\frac{\Gamma \Rightarrow \Delta}{\Sigma, \Gamma \Rightarrow \Delta, \Pi}\ (\text{Weakening, W}) \qquad \frac{\Gamma \Rightarrow \Delta, A \quad A, \Gamma \Rightarrow \Delta}{\Gamma \Rightarrow \Delta}\ (\text{Cut, C})$$

$$\frac{A, B, \Gamma \Rightarrow \Delta}{A \wedge B, \Gamma \Rightarrow \Delta}\ (\wedge\text{L}) \qquad \frac{\Gamma \Rightarrow \Delta, A \quad \Gamma \Rightarrow \Delta, B}{\Gamma \Rightarrow \Delta, A \wedge B}\ (\wedge\text{R})$$

$$\frac{\Gamma \Rightarrow \Delta, A \quad B, \Gamma \Rightarrow \Delta}{A \to B, \Gamma \Rightarrow \Delta}\ (\to\text{L}) \qquad \frac{A, \Gamma \Rightarrow B}{\Gamma \Rightarrow A \to B}\ (\to\text{R})_{\text{S}}$$

$$\frac{\Gamma \Rightarrow \Delta, A}{\neg A, \Gamma \Rightarrow \Delta}\ (\neg\text{L}) \qquad \frac{A, \Gamma \Rightarrow}{\Gamma \Rightarrow \neg A}\ (\neg\text{R})_{\text{S}}$$

$$\frac{A[y/x], \Gamma \Rightarrow \Delta}{\forall x A, \Gamma \Rightarrow \Delta}\ (\forall\text{L}) \qquad \frac{\Gamma \Rightarrow A[z/x]}{\Gamma \Rightarrow \forall x A}\ (\forall\text{R})_{\text{S, VC}}$$

$$\frac{{\sim}A, \Gamma \Rightarrow \Delta \quad {\sim}B, \Gamma \Rightarrow \Delta}{{\sim}(A \wedge B), \Gamma \Rightarrow \Delta}\ ({\sim}\wedge\text{L}) \qquad \frac{\Gamma \Rightarrow \Delta, {\sim}A, {\sim}B}{\Gamma \Rightarrow \Delta, {\sim}(A \wedge B)}\ ({\sim}\wedge\text{R})$$

$$\frac{A, {\sim}B, \Gamma \Rightarrow \Delta}{{\sim}(A \to B), \Gamma \Rightarrow \Delta}\ ({\sim}{\to}\text{L}) \qquad \frac{\Gamma \Rightarrow \Delta, A \quad \Gamma \Rightarrow \Delta, {\sim}B}{\Gamma \Rightarrow \Delta, {\sim}(A \to B)}\ ({\sim}{\to}\text{R})$$

$$\frac{A, \Gamma \Rightarrow \Delta}{{\sim}\neg A, \Gamma \Rightarrow \Delta}\ ({\sim}\neg\text{L}) \qquad \frac{\Gamma \Rightarrow \Delta, A}{\Gamma \Rightarrow \Delta, {\sim}\neg A}\ ({\sim}\neg\text{R})$$

$$\frac{A, \Gamma \Rightarrow \Delta}{{\sim}{\sim}A, \Gamma \Rightarrow \Delta}\ ({\sim}{\sim}\text{L}) \qquad \frac{\Gamma \Rightarrow \Delta, A}{\Gamma \Rightarrow \Delta, {\sim}{\sim}A}\ ({\sim}{\sim}\text{R})$$

$$\frac{{\sim}A[z/x], \Gamma \Rightarrow \Delta}{{\sim}\forall x A, \Gamma \Rightarrow \Delta}\ ({\sim}\forall\text{L})_{\text{VC}} \qquad \frac{\Gamma \Rightarrow \Delta, {\sim}A[y/x]}{\Gamma \Rightarrow \Delta, {\sim}\forall x A}\ ({\sim}\forall\text{R})$$

Here the subscript S indicates the condition of no side formulas, that is, the succedent of the conclusion must consist of only one formula. And VC the *eigenvariable condition*, i.e. the eigenvariable

z must not have free occurrences in the conclusion. Note that the $\sim$-free part of $\mathbf{G}\mathsf{N}$ coincides with $\mathbf{LJ}'$, one of sequent systems for Int.

A formula A is *provable in* $\mathbf{G}\mathsf{N}$, denoted $\mathbf{G}\mathsf{N} \vdash A$, if $\mathbf{G}\mathsf{N} \vdash\ \Rightarrow A$.

For a variant of N, we can obtain its sequent system by adding or omitting the corresponding initial sequents. However for those with D we replace the rule $(\forall\mathrm{R})_{\mathrm{T}}$ instead, by

$$\frac{\Gamma \Rightarrow \Delta, A[z/x]}{\Gamma \Rightarrow \Delta, \forall x A}\ (\forall\mathrm{R})_{\mathrm{VC}}$$

2.2 Kripke-type possible world semantics

Kripke-type semantics for logics in the N-family is a natural extension of that for Int. An N-model, a model for N, is just an Int-model with an additional interpretation I^- (the *falsum* interpretation) which designates what is *falsified*. In contrast, the original interpretation I^+ (the *verum* interpretation) designates what is *verified*.

Let $(M, \leq)$ be a poset, W a non-empty set, and $U : M \to \mathcal{P}W$, satisfying: $U(a) \neq \emptyset$ for all $a \in M$; $a \leq b$ implies $U(a) \subseteq U(b)$. For every predicate symbol p (we assume p is m-ary), we define two interpretations of p at each $a \in M$, denoted by $p^{I^+(a)}$ and $p^{I^-(a)}$, as subsets of $U(a)^m$, satisfying: $a \leq b$ implies $p^{I^+(a)} \subseteq p^{I^+(b)}$ and $p^{I^-(a)} \subseteq p^{I^-(b)}$; $p^{I^+(a)} \cap p^{I^-(a)} = \emptyset$. Then the quintuple $\mathcal{M} = (M, \leq, U, I^+, I^-)$ is said to be an N-*model*.

Given an N-model $\mathcal{M}$, we extend two interpretations I^+ and I^- into two relations between $a \in M$ and a closed formula A, namely $a \models^+ A$ and $a \models^- A$, inductively on the construction of a closed formula A:

$$
\begin{array}{lcl}
a \models^+ p(\underline{u_1}, \ldots, \underline{u_m}) & \iff & (u_1, \ldots, u_m) \in p^{I^+(a)}\ ; \\
a \models^- p(\underline{u_1}, \ldots, \underline{u_m}) & \iff & (u_1, \ldots, u_m) \in p^{I^-(a)}\ ; \\
a \models^+ A \wedge B & \iff & a \models^+ A \ \text{ and } \ a \models^+ B\ ; \\
a \models^- A \wedge B & \iff & a \models^- A \ \text{ or } \ a \models^- B\ ; \\
a \models^+ A \to B & \iff & \text{for every } b \geq a, \ \ b \models^+ A \ \text{ implies } \ b \models^+ B\ ; \\
a \models^- A \to B & \iff & a \models^+ A \ \text{ and } \ a \models^- B\ ; \\
a \models^+ \neg A & \iff & \text{for every } b \geq a, \ \ b \not\models^+ A\ ; \\
a \models^- \neg A & \iff & a \models^+ A\ ; \\
a \models^+ \sim A & \iff & a \models^- A\ ; \\
a \models^- \sim A & \iff & a \models^+ A\ ; \\
a \models^+ \forall x A & \iff & \text{for every } b \geq a \text{ and every } u \in U(b), \ \ b \models^+ A[\underline{u}/x]\ ; \\
a \models^- \forall x A & \iff & \text{for some } u \in U(a), \ \ a \models^- A[\underline{u}/x]\ .
\end{array}
$$

A formula A of N is *valid* in an N-model $\mathcal{M}$, denoted by $\mathcal{M} \models A$, if $a \models^+ \forall \overrightarrow{x} A$ for every $a \in M$, where $\forall \overrightarrow{x} A$ is a universal closure of A. A is *valid*, denoted by $\mathsf{N} \models A$, if A is valid in every N-model. The validity of a sequent $\Gamma \Rightarrow \Delta$ is defined in terms of its formulaic translation $\bigwedge \Gamma \to \bigvee \Delta$.

$a \models^+ A$ or $a \models^- A$ can be read as a *verifies* or *falsifies* A. While whether Heyting's negation holds is reduced (through *reasoning*) to the verum interpretation I^+, strong negation is not, being reduced to the falsum interpretation I^- which is primitive and independent from I^+.

Kripke-type semantics for the logics other than N is obtained by the following modifications:

- D Add the condition that the domain $U : M \to \mathcal{P}W$ is a constant map.
- L Add the condition that $(M, \leq)$ is linearly ordered.
- O Add the condition that for every $a \in M$ and every closed formula A, there exists $b \geq a$ where either $b \models^+ A$ or $b \models^- A$ holds.
- P Omit the condition $p^{I^+(a)} \cap p^{I^-(a)} = \emptyset$.

Several examples of models for the N-family are presented in Fig. 2. A circle designates a possible world $a \in M$, varying its size according to the size of its domain $U(a)$. Each circle has two colored parts: the upper designates the formulas which are verified in it, and the lower those which are falsified.

The following lemmas and the soundness theorem are obtained easily by induction.

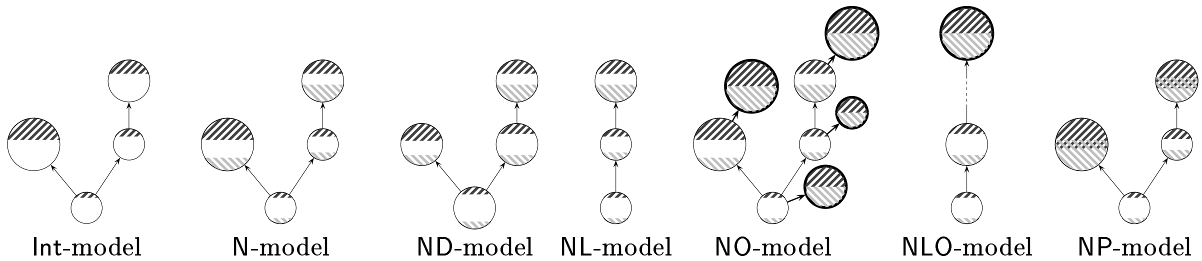


Fig. 2: Kripke models

Lemma 2.1 (Heredity) *Let A be a closed formula of* N *(or equivalently* N[D][L][O][P]*), $\mathcal{M} = (M, \leq, U, I^+, I^-)$ be an* N[D][L][O][P]*-model, $a, b \in M$ and $a \leq b$. Then $a \models^+ A$ (or $a \models^- A$) implies $b \models^+ A$ (or $b \models^- A$, respectively).*

Lemma 2.2 *Let A be a closed formula of* N*, $\mathcal{M} = (M, \leq, U, I^+, I^-)$ be an* N[D][L][O]*-model and $a \in M$. Then it is impossible that both $a \models^+ A$ and $a \models^- A$ hold.*

Theorem 2.3 (Kripke soundness of **G**N[D][L][O][P]) *If* **G**N[D][L][O][P] $\vdash \Gamma \Rightarrow \Delta$*, then* N[D][L][O][P] $\models \Gamma \Rightarrow \Delta$*, respectively.*

3 Kripke completeness of **G**N[D][L][P]

We are now to consider completeness, using the tree-sequent method. First the proof for **G**N is presented, and others are obtained by the slight modifications designated thereafter. Omitting $\sim$, the arguments provide simpler proofs for completeness of Int, CD and LC.

Definition 3.1 (Tree-sequent of **T**N) A *tree-sequent* $\mathcal{T}$ of **T**N is a finite labelled tree, each node a of which is associated with a sequent $\Gamma_a \Rightarrow \Delta_a$ of **G**N and a finite set of variables α_a, denoted by $(a : \Gamma_a \overset{\alpha_a}{\Rightarrow} \Delta_a)$, satisfying the following conditions:

1. Let 0 be the root of $\mathcal{T}$, and $a_0 (= 0), a_1, \ldots, a_n$ be an arbitrary path in $\mathcal{T}$, and $(a_i : \Gamma_i \overset{\alpha_i}{\Rightarrow} \Delta_i)$ for each $i \in [0, n]$. Then $\alpha_0, \alpha_1, \ldots, \alpha_n$ are disjoint. The (disjoint) union $\alpha_0 \cup \alpha_1 \cup \cdots \cup \alpha_n$ is said to be the set of *available variables* at the node a_n.

2. Every free variable in the sequent associated to a is available at a.

In other words, for $(a : \Gamma_a \overset{\alpha_a}{\Rightarrow} \Delta_a)$, α_a is the set of variables which are available at a for the first time in tracing from the root. Omitting the condition 2., we obtain the definition of a *pre-tree-sequent* of **T**N, which is abbreviated as a pTS and will emerge as a subtree of a TS.

In what follows a TS is denoted using [,] and |. $[\Gamma \overset{\alpha}{\Rightarrow} \Delta \mid \mathcal{T}_1 \ldots \mathcal{T}_k]$ is a TS with its root $\Gamma \overset{\alpha}{\Rightarrow} \Delta$ followed by its subtrees $\mathcal{T}_1, \ldots, \mathcal{T}_k$.

Definition 3.2 (Tree-sequent system **T**N) ‖

$$\frac{}{\cdots[A \overset{\alpha}{\Rightarrow} A \mid \cdots}\ (\mathrm{Id}) \qquad \frac{}{\cdots[A, \sim A \overset{\alpha}{\Rightarrow} \mid \cdots}\ (\mathrm{Fal})$$

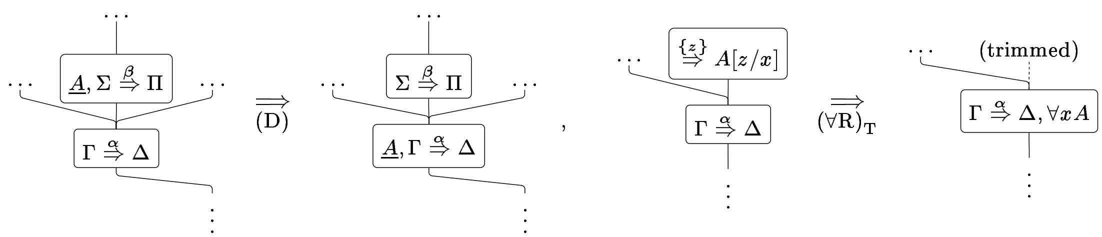
$$\frac{\cdots[\Gamma \overset{\alpha}{\Rightarrow} \Delta \mid \cdots}{\cdots[\Sigma, \Gamma \overset{\alpha}{\Rightarrow} \Delta, \Pi \mid \cdots}\ (\mathrm{W}) \qquad \frac{\cdots[\Gamma \overset{\alpha}{\Rightarrow} \Delta \mid \cdots[A, \Sigma \overset{\beta}{\Rightarrow} \Pi \mid \cdots]\cdots]\cdots}{\cdots[A, \Gamma \overset{\alpha}{\Rightarrow} \Delta \mid \cdots[\Sigma \overset{\beta}{\Rightarrow} \Pi \mid \cdots]\cdots]\cdots}\ (\mathrm{Drop,\ D})$$

($\wedge$L), ($\wedge$R), ($\to$L), ($\neg$L), ($\forall$L), ($\sim\wedge$L), ($\sim\wedge$R), ($\sim\to$L), ($\sim\to$R), ($\sim\neg$L), ($\sim\neg$R), ($\sim\sim$L), ($\sim\sim$R), $(\sim\forall\mathrm{L})_{\mathrm{VC}}$ and ($\sim\forall$R) are of the form of (W), applying the corresponding rule of **G**N to one node.

$$\frac{\cdots[\Gamma \overset{\alpha}{\Rightarrow} \Delta \mid \cdots[A \overset{\emptyset}{\Rightarrow} B]\cdots]\cdots}{\cdots[\Gamma \overset{\alpha}{\Rightarrow} \Delta, A \to B \mid \cdots}\ (\to\mathrm{R})_{\mathrm{T}} \qquad \frac{\cdots[\Gamma \overset{\alpha}{\Rightarrow} \Delta \mid \cdots[A \overset{\emptyset}{\Rightarrow}\ ]\cdots]\cdots}{\cdots[\Gamma \overset{\alpha}{\Rightarrow} \Delta, \neg A \mid \cdots}\ (\neg\mathrm{R})_{\mathrm{T}} \qquad \frac{\cdots[\Gamma \overset{\alpha}{\Rightarrow} \Delta \mid \cdots[\ \overset{\{z\}}{\Rightarrow} A[z/x]\,]\cdots]\cdots}{\cdots[\Gamma \overset{\alpha}{\Rightarrow} \Delta, \forall x A \mid \cdots}\ (\forall\mathrm{R})_{\mathrm{T}}$$

‖ **T**N[D][L][P] enjoys one remarkable property; it is cut-free. Although the fact may imply some other proof-theoretical results, they are beyond our scope.

Here a tree-structure and undisplayed nodes are arbitrary; (Id) is read as every TS which has a node of the form $A \overset{\alpha}{\Rightarrow} A$ is an initial TS. In the derivation rules, the conclusion and the hypothesis (or hypotheses) share their undisplayed tree-structures and nodes; (W) is adding formulas to one node. VC is for the condition that there are no occurrences of the eigenvariable z in *any* node of the conclusion. The rule (D) and those with a subscript T (*trim a leaf* of a TS) are unique to TS system, as presented below:



Definition 3.3 (Counter model for tree-sequent) Let $\mathcal{M} = (M, \leq, U, I^+, I^-)$ be an N-model, $U : M \to \mathcal{P}W$, and $\mathcal{T}$ a TS of $\mathbf{T}\mathsf{N}$. $\mathcal{M}$ is said to be a *counter model for* $\mathcal{T}$ if: the tree-structure of $\mathcal{T}$ can be embedded in $(M, \leq)$; the set of all variables can be embedded in W; for each node $(a : \Gamma_a \overset{\alpha_a}{\Rightarrow} \Delta_a)$ of $\mathcal{T}$, $a \models^+ A[\underline{\overrightarrow{x}}/\overrightarrow{x}]$ for each $A \in \Gamma_a$, and $a \not\models^+ B[\underline{\overrightarrow{y}}/\overrightarrow{y}]$ for each $B \in \Delta_a$.**

Lemma 3.4 (Kripke completeness of $\mathbf{T}\mathsf{N}$) *Suppose* $\mathbf{T}\mathsf{N} \not\vdash \mathcal{T}$ *and there is at least one variable available at the root of* $\mathcal{T}$. *Then* $\mathcal{T}$ *has a counter* N*-model.*

Proof. Just as completeness proofs based on tableaux systems do, extend $\mathcal{T}$ into a *saturated* TS $\mathcal{T}_\infty$, which induces a counter-model $\mathcal{M}$. For example, for a node $\Gamma \overset{\alpha}{\Rightarrow} \Delta$ with $A \to B$ in Δ, make a new successor (a leaf) labelled with $A \overset{\emptyset}{\Rightarrow} B$. In $\mathcal{M}$, the domain $U(a)$ at a world a consists of all the variables available at a in $\mathcal{T}_\infty$. For the precise definitions and procedures, see (Hasuo and Kashima, 2003). ∎

Definition 3.5 (Formulaic translation of TS) Let $\mathcal{T}$ be a pTS of N. The *formulaic translation* of $\mathcal{T}$, denoted by $\mathcal{T}^f$, is defined inductively on the height of $\mathcal{T}$:

$$[\Gamma \overset{\alpha}{\Rightarrow} \Delta \mid \mathcal{T}_1 \dots \mathcal{T}_m]^f :\equiv \forall \overrightarrow{\alpha} \big((\textstyle\bigwedge \Gamma) \to (\textstyle\bigvee \Delta) \vee \mathcal{T}_1^f \vee \cdots \vee \mathcal{T}_m^f \big)$$

Lemma 3.6 *If* $\mathbf{T}\mathsf{N} \vdash \mathcal{T}$, *then* $\mathbf{G}\mathsf{N} \vdash \mathcal{T}^f$.

Proof. By induction. Again see (Hasuo and Kashima, 2003) for detailed descriptions. ∎

Theorem 3.7 (Kripke completeness of $\mathbf{G}\mathsf{N}$) *If* $\mathsf{N} \models A$, *then* $\mathbf{G}\mathsf{N} \vdash A$.

Proof. Let $\mathbf{G}\mathsf{N} \not\vdash A$ and $\mathcal{T} :\equiv [\overset{\alpha}{\Rightarrow} A]$. Then $\mathbf{T}\mathsf{N} \not\vdash \mathcal{T}$ by Lemma 3.6, and Lemma 3.4 yields the existence of a counter model for $\mathcal{T}$, hence that for A. ∎

The TS method is also applicable to other logics in $\mathbf{G}\mathsf{N}[\mathsf{D}][\mathsf{L}][\mathsf{P}]$ by making some modifications which seem quite natural. For those with D, TSs do not involve availability of variables. For those with L, TSs are not trees but just finite sequences of sequents (recall that an NL-model is a *linearly-ordered* N-model!), and the rules with a subscript T are replaced; e.g. $(\forall \mathrm{R})_{\mathrm{T}}$ is replaced by

$$\frac{\mathcal{T}_1 \quad \mathcal{T}_2 \quad \cdots \quad \mathcal{T}_k}{\cdots \mid \Gamma_1 \overset{\alpha_1}{\Rightarrow} \Delta_1, \forall x A \mid \Gamma_2 \overset{\alpha_2}{\Rightarrow} \Delta_2 \mid \cdots \mid \Gamma_k \overset{\alpha_k}{\Rightarrow} \Delta_k} \, (\forall \mathrm{R})_{\mathrm{K}}$$

where for each $i \in [1, k]$,

$$\mathcal{T}_i \equiv \quad \cdots \mid \Gamma_1 \overset{\alpha_1}{\Rightarrow} \Delta_1 \mid \cdots \mid \Gamma_i \overset{\alpha_i}{\Rightarrow} \Delta_i \mid \quad \overset{\{z\}}{\Rightarrow} A[z/x] \, \text{(inserted)} \mid \cdots \mid \Gamma_k \overset{\alpha_k}{\Rightarrow} \Delta_k \quad .$$

Theorem 3.8 (Kripke completeness of $\mathbf{G}\mathsf{N}[\mathsf{D}][\mathsf{L}][\mathsf{P}]$) *If* $\mathsf{N}[\mathsf{D}][\mathsf{L}][\mathsf{P}] \models A$, *then* $\mathbf{G}\mathsf{N}[\mathsf{D}][\mathsf{L}][\mathsf{P}] \vdash A$.

**Every TS derived in $\mathbf{T}\mathsf{N}$ has no counter models, which justifies such unfamiliar rules as (D) or $(\forall \mathrm{R})_{\mathrm{T}}$.

4 Kripke completeness of GN[D][L]O

4.1 Proof by an embedding of Cl – for GN[L]O

As stated in the introduction, completeness of **G**N[D][L]O can hardly be shown by a simple application of the TS method. The first proof, given in this subsection, utilizes an embedding of Cl in **G**N[L]O, for obtaining *omniscient* worlds where every closed formula is either verified or falsified. In the following by $A_{\sim\neg}$ we denote a formula of N obtained by replacing an arbitrary number of $\neg$'s in A by $\sim$.

Lemma 4.1 (Embedding of **LK** in **G**N[D][L]O) *The following are all equivalent:*

1. $\mathbf{LK} \vdash \Gamma_\neg \Rightarrow \Delta_\neg$;
2. $\mathbf{G}\mathsf{N[D][L]O} \vdash \Gamma_{\sim\neg}, \sim\Delta_{\sim\neg} \Rightarrow$;
3. $\mathbf{G}\mathsf{N[D][L]O} \vdash \Gamma_{\sim\neg}, \neg\Delta_{\sim\neg} \Rightarrow$.

Proof. We assume that the logical connectives of Cl are $\wedge$, $\rightarrow$, $\neg$ and $\forall$, in order to make correspondence with N[D][L]O.
[1. $\Rightarrow$ 2.] By induction on derivation in **LK**, restricting initial sequents to those of the form $p(\overrightarrow{x}) \Rightarrow p(\overrightarrow{x})$. Here the following derivations which are possible in **G**N[D][L]O play important roles:

$$\frac{}{\neg\sim A, \neg A \Rightarrow}\ (\mathrm{Om1}) \qquad \frac{\sim A, \Gamma \Rightarrow}{\neg A, \Gamma \Rightarrow}\ (\mathrm{Om2}) \qquad \frac{}{A \rightarrow B \Rightarrow \neg\neg(\sim A \vee B)}\ (\mathrm{Om3})$$

[2. $\Rightarrow$ 1.] Semantically. An Cl-model induces an N[D][L]O-model, and use completeness of **LK**.
[2. $\Leftrightarrow$ 3.] Obvious by (Om2) above and that $\mathbf{G}\mathsf{N[D][L]O} \vdash \sim A \Rightarrow \neg A$. ∎

Theorem 4.2 (Kripke completeness of **G**N[L]O) *If* N[L]O $\models A$*, then* **G**N[L]O $\vdash A$.

Proof. We define TS systems **T**N[L]O for N[L]O as they have the cut rule (C). Then a TS which has a **G**N[L]O-provable sequent as a node is also provable in **T**N[L]O (Fact 1). For an **LK**-consistent infinite sequent, i.e. one whose finite subsequent is always unprovable, we can construct a counter Cl-model by increasing variables twofold (Fact 2). An omniscient possible world a_g, where $p^{I^+(a_g)} \cup p^{I^-(a_g)} = U(a_g)^m$, is induced by a Cl-model (Fact 3). Using these facts and the above lemma, we can construct a counter N[L]O-model for an unprovable formula A, as shown in Fig. 3. Here $\mathcal{T}_\infty$ is a **T**N[L]-saturated TS obtained by extending $[\overset{\alpha}{\Rightarrow} A]$. ∎

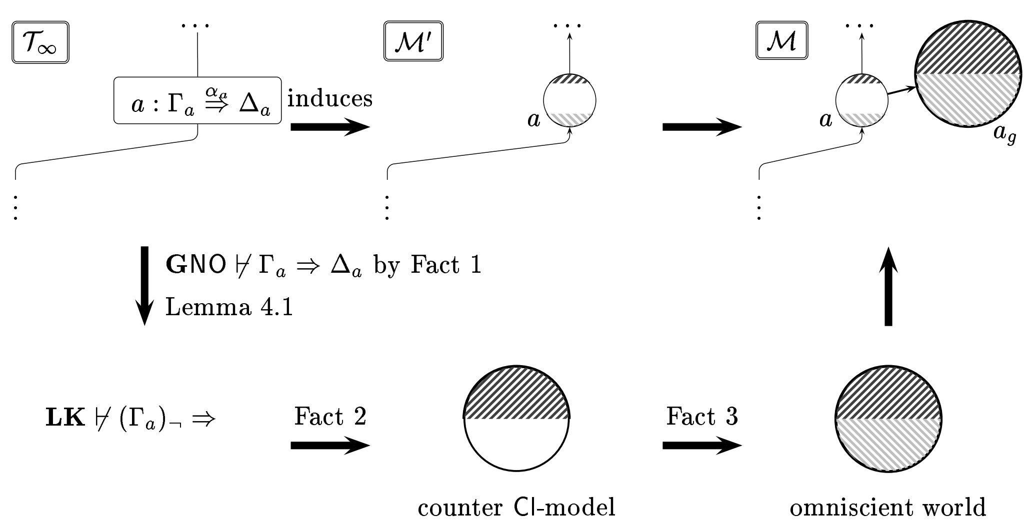


Fig. 3: Sketch of the proof

4.2 Proof by the TSg method – for $\mathbf{GN}\mathsf{[D][L]O}$

The method in the previous subsection is not applicable to $\mathbf{GN}\mathsf{D[L]O}$. In the method we construct omniscient possible worlds *after we have finished* extending an unprovable formula into an *infinite* TS, hence it becomes necessary to increase variables, which results in extended domains.

To avoid the problem, we introduce a method utilizing a *tree-sequent with guardians*, TSg in short, which is an extension of a TS. Here we extend both a TS and seeds of omniscient worlds *simultaneously*.

A TSg is a TS each node of which has an extra sequent, called the *guardian*, hence a node of a TSg can be denoted by $(a : \Gamma_a \overset{\alpha_a}{\Rightarrow} \Delta_a \uparrow \Sigma_a \overset{\beta_a}{\Rightarrow} \Delta_a)$. The guardian of a is a seed of the omniscient world a_g which will be assigned to a.

The proofs of completeness are just like the TS-based ones for $\mathbf{GN}\mathsf{[D][L][P]}$. First we introduce a TSg system, then its completeness is easily obtained extending an unprovable TSg into a saturated one, which induces a counter model. Using the formulaic translation of a TSg, we can conclude completeness of Gentzen-style system. For example, a TSg system has initial TSgs $\cdots[\Gamma \overset{\alpha}{\Rightarrow} \Delta \uparrow \overset{\beta}{\Rightarrow} A \vee {\sim}A \mid \cdots$ (gOm), and the formulaic translation is defined by

$$[\Gamma \overset{\alpha}{\Rightarrow} \Delta \uparrow \Sigma \overset{\beta}{\Rightarrow} \Pi \mid \mathcal{G}_1 \ldots \mathcal{G}_m]^f :\equiv \forall\vec{\alpha}\Big((\bigwedge \Gamma) \to (\bigvee \Delta) \;\vee\; \forall\vec{\beta}\neg\neg\Big((\bigwedge \Sigma) \to (\bigvee \Pi) \Big) \;\vee \mathcal{G}_1^f \vee \cdots \vee \mathcal{G}_m^f \Big) .$$

Again and again many details are left in (Hasuo and Kashima, 2003).

THEOREM 4.3 (KRIPKE COMPLETENESS OF $\mathbf{GN}\mathsf{[D][L]O}$) *If* $\mathsf{N[D][L]O} \models A$, *then* $\mathbf{GN}\mathsf{[D][L]O} \vdash A$.

5 Concluding remarks and future work

Completeness of $\mathbf{GN}\mathsf{[D][L]OP}$ Kripke completeness of those with both O and P remain unproved in this paper, as stated in the introduction. Actually its proof cannot be done by our methods; the key is a well-known axiom $\forall x\neg\neg A \to \neg\neg\forall x A$, called the *double negation shift* (DNS) or *Kuroda's axiom*. The DNS is a theorem of those logics with O but without P.

$$\cfrac{\cfrac{\cfrac{\cfrac{\cfrac{}{A[z/x], {\sim}A[z/x] \Rightarrow}\,(\text{Fal})}{\forall x\neg\neg A, {\sim}A[z/x] \Rightarrow}\,(\neg\text{R}), (\neg\text{L}), (\forall\text{L})}{\forall x\neg\neg A, {\sim}\forall x A \Rightarrow}\,({\sim}\forall\text{L})}{\forall x\neg\neg A, \neg\forall x A \Rightarrow}\,(\text{Om2})}{\Rightarrow \forall x\neg\neg A \to \neg\neg\forall x A}\,(\neg\text{R}), (\to\text{R})$$

On the other hand, the DNS is not provable in even the strongest system with both O and P, i.e. $\mathbf{GN}\mathsf{DLOP}$, since it has a counter model $\mathcal{M} = (M, \leq, U, I^+, I^-)$ constructed as follows: $(M, \leq) = (\mathbf{N}, \leq)$, $U(n) = \mathbf{N}$ for all $n \in \mathbf{N}$, p a unary predicate symbol, $p^{I^+(n)} = \{0, 1, 2, \ldots, n\}$ and $p^{I^-(n)} = \mathbf{N}$. Then $\mathcal{M} \not\models \forall x\neg\neg p(x) \to \neg\neg\forall x p(x)$.

The intermediate logic MH, which is Int plus the DNS, is characterized by the class of Int-models which satisfy: $(*)$ for each possible world a, there exists $b \geq a$ which is maximal (Gabbay, 1981). To prove completeness of $\mathbf{GN}\mathsf{[D][L]OP}$ we must construct a counter model for the DNS, which we cannot by our methods since every counter model constructed therein satisfy the property $(*)$ (omniscient worlds are made to be maximal).

The logic (NO plus $\neg A \vee \neg\neg A$) The intermediate logic KC (Int plus $\neg A \vee \neg\neg A$, the weak law of excluded middle) is characterized by the class of directed Int-models, that is, if $a \leq b$ and $a \leq c$ then there exists d such that $b \leq d$ and $c \leq d$. In fact for the unquantified case the condition can be strengthened that a model has its maximum; for the quantified case it is too strong since the DNS is not a theorem of KC.

Here regarding that the DNS is a theorem of NO, it seems rather possible that the logic NOW (NO plus $\neg A \vee \neg\neg A$) is characterized by the class of N-models with the maximum which is omniscient (Fig. 4)[††]

Such development of information states seems to be at least as common as NLO-models, hence the author finds a considerable amount of interest in the problem of whether the correspondence holds or not.

[††]Kripke completeness of the logic (Int + $\neg A \vee \neg\neg A$ + the DNS + the axiom of constant domain) is already given in (Ono, 1987), which may be circumstantial positive evidence.

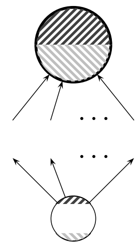


Fig. 4: NOW-model?

Acknowledgement

The author would like to thank Professor Ryo KASHIMA most sincerely for his supervision. He is also indebted for invaluable comments and information to Professors Norihiro KAMIDE, Izumi TAKEUTI, Takeshi YAMAGUCHI, Katsumi SASAKI, Tatsuya SHIMURA, Hiroakira ONO and the anonymous referees.

References

Akama, Seiki. 1997. Tableaux for logic programming with strong negation. In Didier Galmiche, editor, *Automated Reasoning with Analytic Tableaux and Related Methods, International Conference, TABLEAUX'97*, volume 1227 of *Lecture Notes in Computer Science*, pages 31–42. Springer-Verlag, May.

Almukdad, Ahmad and David Nelson. 1984. Constructible falsity and inexact predicates. *The Journal of Symbolic Logic*, 49(1):231–233.

Corsi, Giovanna. 1992. Completeness theorem for Dummett's LC quantified and some of its extensions. *Studia Logica*, 51:317–335.

Corsi, Giovanna and Silvio Ghilardi. 1989. Directed frames. *Archive for Mathematical Logic*, 29:53–67.

Gabbay, Dov M. 1981. *Semantical Investigations in Heyting's Intuitionistic Logic*, volume 148 of *Synthese Library*. D. Reidel.

Goranko, Valentine. 1985. The craig interpolation theorem for propositional logics with strong negation. *Studia Logica*, 44:291–317.

Görnemann, S. 1971. A logic stronger than intuitionism. *The Journal of Symbolic Logic*, 36:249–261.

Grzegorczyk, A. 1964. A philosophically plausible formal interpretation of intuitionistic logic. *Indagationes Mathematicae*, 26:596–601.

Hasuo, Ichiro and Ryo Kashima. 2003. Kripke completeness of first-order constructive logics with strong negation. Available at `http://www.is.titech.ac.jp/~hasuo2`.

Herre, Heinrich and David Pearce. 1992. Disjunctive logic programming, constructivity and strong negation. In *Logic in AI. Proceedings of the European Workshop JELIA 92*, volume 633 of *Lecture Notes in Artificial Intelligence*. Springer-Verlag.

Kashima, Ryo. 1999. Sequent calculi of non-classical logics – Proofs of completeness theorems by sequent calculi (in Japanese). In *Proceedings of Mathematical Society of Japan Annual Colloquium of Foundations of Mathematics*, pages 49–67.

Kracht, Marcus. 1998. On extensions of intermediate logics by strong negation. *Journal of Philosophical Logic*, 27:49–73.

Markov, A. A. 1950. Konstruktivnaja logika. *Uspehi Matematičeskih Nauk*, 5:187–188.

Nelson, David. 1949. Constructible falsity. *The Journal of Symbolic Logic*, 14:16–26.

Ono, Hiroakira. 1987. Model extension theorem and Craig's interpolation theorem for intermediate predicate logics. *Reports on Mathematical Logic*, 21:55–67.

Pearce, David and Gerd Wagner. 1990. Reasoning with negative information, I: Strong negation in logic programs. In *Language, Knowledge and Intentionality (Acta Philosophica Fennica 49)*, pages 405–439.

Pearce, David and Gerd Wagner. 1991. Logic programming with strong negation. In Peter Schroeder-Heister, editor, *Extensions of Logic Programming*, volume 475 of *Lecture Notes in Artificial Intelligence*, pages 311–326. Springer-Verlag.

Priest, Graham and Richard Routley. 1984. Introduction: Paraconsistent logics. *Studia Logica*, 43:3–16.

Sendlewski, Andrzej. 1984. Some investigations of varieties of N-lattices. *Studia Logica*, 43:257–280.

Sendlewski, Andrzej. 1990. Nelson algebras through Heyting ones. *Studia Logica*, 49:106–126.

Thomason, R. H. 1969. A semantical study of constructive falsity. *Zeitschrift für mathematische Logik und Grundlagen der Mathematik*, 15:247–257.

van Dalen, D. 1986. Intuitionistic logic. In D. M. Gabbay and F. Guenthner, editors, *Handbook of Philosophical Logic III*, volume 166 of *Synthese Library*. D. Reidel, pages 225–340.

Wagner, Gerd. 1994. *Vivid Logic. Knowledge-Based Reasoning with Two Kinds of Negation*, volume 764 of *Lecture Notes in Artificial Intelligence*. Springer-Verlag.

Wansing, Heinrich. 1993. *The Logic of Information Structures*, volume 681 of *Lecture Notes in Artificial Intelligence*. Springer-Verlag.

Discontinuous Verb-Object Compounds in Cantonese

Natalie So-Sum Yu
The University of Hong Kong
Department of Linguistics
ynatalie@hkusua.hku.hk

Abstract

This paper explores the nature of a kind of construction in the Chinese language that has been referred to as Verb-Object Compound (VOC). A VOC is made up of a monosyllabic verbal element (V) and a monosyllabic nominal element (N). Morphologically, the two constituents are bound to each other and they go together like a fused form. This can be seen from the fact that VOCs are usually treated as lexical entries in dictionaries and are acquired as wholes. Syntactically, however, the VOC allows insertion of extraneous materials between the two constituents. This makes it appear to be a phrase. Its dual status is exactly the root of many discussions in the past. The question of how to represent an expression which has a syntactically complex structure with respect to certain phenomena and yet is syntactically simple regarding other phenomena has always been a problematic issue for many theories of syntax. In this paper, I intend to examine Cantonese VOCs from a new perspective set out from the framework of Lexical-Functional Grammar (LFG) (Bresnan 1982, Bresnan 2001, Kaplan and Bresnan 1989). Analyzing the VOC within the LFG framework which factors the grammatical information of an expression into role, category and function, it is believed that the representation of this type of peculiar construction can be accommodated.

Introduction

The literature concerning this type of construction in the Chinese language mostly focuses on the issue of what criteria to use in determining the status of the Verb-Object Forms and much of the studies were done on Mandarin data. The most prominent criteria used in various discussions (Chao 1968: 415, Li and Thompson 1981: 73, Packard 2000: 106-124) include the separability of the constituents, the idiomaticity of meaning and the bound status of the composing elements. In this regard, rather than devoting efforts to the discussion of the validity of these criteria (which has long been a controversial issue in past decades and still now attracts many arguments among linguists), I intend to take a different perspective set in the framework of Lexical-Functional Grammar (LFG) to investigate the Cantonese VOCs.

In Section 1, I will give a brief introduction to the nature of the Verb-Object Forms and the nature of the controversy around this type of construction. In Section 2, I will argue for the dismissal of the object status of the nominal constituent, leading to the further establishment of the unity of the Verb-Object Forms in Section 3 where the difference between the 'VOC phenomenon' and noun incorporation (NI) will also be pointed out . Finally, in Section 4, I shall illustrate how LFG can deal with this type of lexical discontinuity.

1 Verb-Object Forms in the Literature

The difficulty in defining the notion of 'word' in Chinese is salient for a number of reasons. First and foremost, Chinese is not written with word delimiters so segmenting a sentence into 'words' is not a natural task even for a native speaker. Second, Chinese has little inflectional morphology to ease word identification. Third, many monosyllabic morphemes that previously were able to stand alone in non-modern Chinese have become bound in modern Chinese (Chao 1968: 138-139). The influence of non-modern Chinese makes it difficult to draw the line between bound morphemes and free morphemes, the notion which could otherwise have been very useful for deciding word boundaries.

The Chinese Verb-Object Compound (VOC) is one of the prevalent forms that can readily reflect this difficulty and it has been the subject of examination on the issue of wordhood in the Chinese language. A VOC, usually having the grammatical category as a verb[1], is made up of a monosyllabic verbal element (V)

[1] Although the majority of VOCs falls into the grammatical category of verbs, there are VOCs that can be nouns or adjectives which are totally inseparable and are not within the scope of this paper.

and a monosyllabic nominal element (N). Superficially, the V acts as the predicate of the whole construction and the N plays the objective role to the predicate in the construction. In Li and Thompson's (1981: 73) view, '[t]he verb-object compound, as its name indicates, is composed of two constituents having the syntactic relation of a verb and its direct object'. As I shall show later, the nominal is not the object of the verbal after all; it is actually a non-argument and should be viewed as one of the component parts of the whole predicate.

The source of dispute among linguists in past works is the lexical (or phrasal) status of the Chinese VOCs, Mandarin VOCs to be specific. For instance, one may consider [2]***擔心 dan1xin1*** *carry-heart* 'to worry about' as a lexical word. However, it can appear like a syntactic phrase as in ***擔****了三年的****心*** ***dan1*** *le san1 lian2 de* ***xin1*** *carry-PERF*[3]*-three-years-DE-heart* 'has/have worried for three years'. An example of a construction of the same type in Cantonese would be ***游水 jau4seoi2*** *swim-water* 'swim', which can also be separated by a duration phrase as ***游***咗一個鐘頭***水*** ***jau4*** *zo2 jat1 go3 zung1 tau4* ***seoi2*** *swim-PERF-one-CL-hour-water* 'swam for an hour'. ***冲涼 cung1 loeng4*** *wash-bath* 'to have a bath', ***兜風 dau1 fung1*** *wrap up-wind* 'stroll around' are also examples of Cantonese VOCs. The VOC is peculiar in status in the sense that it is often treated as a word since the two constituting elements always go together like a fused form, yet it also allows some other elements to be inserted in the middle, making it appear to be a phrase. It is at this point that different opinions on the status of the VOC come in. Some linguists suggest that these VOCs are words and get ionized when the two components are separated (Chao 1968, Packard 2000). The opposite position claims that they come from syntactic phrases and become lexicalized at a later stage (Huang 1983, Li and Thompson 1981). However, what concerns us here is not the issue that whether VOCs get ionized first or lexicalized first, but the evidence suggesting that they still act as a unit even though they are separated by the insertion of some other lexical items.

2 Syntactic Tests

In the following, I shall use two syntactic tests to prove that the nominal is not the object in the construction and no subordinate relationship exists between the verbal and the nominal. These syntactic tests involve the use of ***將 zoeng1 'ZOENG'*** in test 1 and the ***passive marker 俾 bei2 'PASS'***[4] in test 2.

2.1 Test with 將 zoeng1 'ZOENG'

The first test involves the use of 將 *zoeng1* 'ZOENG' which is used to mark objects or patients/themes in a sentence. If the claim is that the V and the N of a VOC are constituents of an entire complex predicate and there is no governing relationship between V and N, then it should not be possible to mark the N as the object by using 將 *zoeng1* 'ZOENG'. This turns out to be true as indicated by the ungrammaticality of the following expressions:

(1) *將水游咗 (游水 *jau4seoi2* 'swim')[5]
zoeng1 seoi2 jau4 zo2
ZOENG-water-swim-PERF

(2) *將風兜咗 (兜風 *dau1fung1* 'stroll around')
zeong1 fung1 dau1 zo2
ZOENG-wind-wrap up-PERF

[2] All Verb-Object Forms in this paper are italicized for visual distinction from verb phrases which are underlined. Both of the Mandarin data and the Cantonese data have a four-part glossing scheme: the first part are the Chinese characters, the second part is the romanization, the third part is the morpheme-for-morpheme translation and the fourth part is the free translation.
[3] PERF - perfect aspect
[4] PASS - passive marker
[5] The VOCs involved in illustrations are shown in brackets.

2.2 Test with the passive marker 俾 bei2 'PASS'

The second test involves the use of the passive marker 俾 *bei2*. Passive constructions in Cantonese can be marked by 俾 *bei2* 'PASS' which occurs preverbally. By the same token, if the nominal in a VOC can be the grammatical object, it should also be able to occur as the 俾 *bei2* object in the passive construction. However, this is not the case, as expressions (5) and (6) are unacceptable.

(5) *水俾我游咗 (游水 *jau4seoi2* 'swim')
seoi2 bei2 ngo5 jau4 zo2
water-PASS-1.SG-swim-PERF

(6) *風俾我兜咗 (兜風 *dau1fung1* 'stroll around')
fung1 bei2 ngo5 dau1 zo2
wind-PASS-1.SG-wrap up-PERF

The tests capture the generality that the object can be made more prominent through the transformation frames. The ungrammaticality of the transformed sentences is thus a reflection of the incapability of the nominal constituent to act as the object to the verbal. This means that the verbal constituent cannot be a predicate subcategorizing for the nominal constituent as the object.

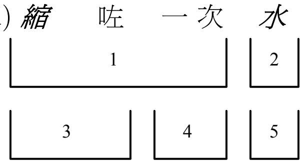
3 The Unity of Verb-Object Forms

Having dismissed the object status of the nominal, I now go on to further elucidate the unitary status, despite the discontinuity of the two constituents, of the Cantonese VOC by citing more evidence in the following sub-sections.

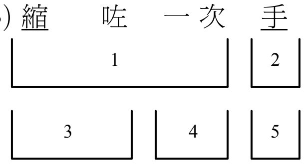
3.1 The Relationship between the Verbal and the Nominal

By examining the relationship between the internal elements of the expanded Verb-Object Form, the unity of the V and the N forming a predicate becomes more obvious. Let me take the Verb-Object Forms ***縮水 suk1seoi2*** *shrink-water* 'to shrink through getting wet' and ***入罪 jap6 zeoi6*** *enter-guilt* 'to convict' as illustrations to this point. These two Verb-Object Forms are taken to compare with two verb phrases 縮手 *suk1sau2* *withdraw-hand* 'withdraw one's hands' and 入屋 *jap6 uk1* *enter-house* 'enter the house' having the homophonous verbal constituent as the two Verb-Object Forms. Through the comparison, the non-subordinate relationship between the V and the N can be revealed.

(A) ***縮*** 咗 一次 ***水*** ***suk1*** *zo2* *jat1* *ci3* ***seoi2***
shrink PERF one time water
'shrunk through getting wet once'

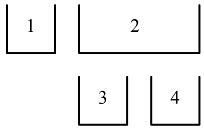


(B) 縮 咗 一次 手 *suk1* *zo2* *jat1* *ci3* *sau2*
withdraw PERF one time hand
'withdrew one's hand once'



The surface structure of the above two sentences is exactly the same. They have an identical constituent structure. The difference lies in the underlying relationship between the constituents. In B, the nominal (5) 手 *sau2* 'hand' is the entity toward which the act described by the verbal element (3) 縮咗 *suk1zo2* 'withdrew' directs. However, in A, the nominal (5) 水 *seoi2* 'water' cannot be acted upon by the behaviour described by the verbal (3) 縮咗 *suk1zo2* 'shrunk'. The same analysis can be applied to the following expression pairs ***入罪 jap6 zeoi6*** *enter-guilt* 'to convict' and 入屋 *jap6 uk1* *enter-house* 'enter the house'.

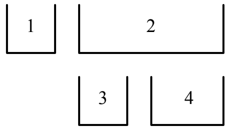
(C) 入　佢　***罪***　　***jap6***　*koei5*　***zeoi6***



enter　3.SG[6]　guilt

'to convict him/her'

(D) 入　佢　屋企　　*jap6*　*koei5*　*uk1kei5*



enter　3.SG　house

'to enter his/her house'

In D, the verbal (1) 入 *jap6* 'to enter' can come into collocation with the nominal (4) 屋企 *uk1kei5* 'house'. However, in C, the nominal (4) 罪 *zeoi6* 'guilt' cannot be an entity for one to carry out the action described by the verbal (1) 入 *jap6* 'to enter'.

By revealing the underlying relationship between the constituents, we can distinguish the difference between the two types of expressions having an identical surface structure. In B and D, there is a predicting relationship between the verbal and the nominal. However, this relationship does not exist in A and B.

3.2 Modification by Classifiers

The noun in a verb phrase can be preceded and modified by a nominal classifier (indicating quantity) or a verbal classifier (indicating frequency) since it is the object of the phrase which is syntactically and semantically modifiable. For instance, 擔水 *daam1 seoi2 carry-water* 'carry water' can be modified by verbal classifiers like 一次 *jat1 ci3 one-time* 'once' to become 擔咗一次水 *daam1 zo2 jat1 zi3 seoi2 carry-PERF-one-time-water* 'carried water once' and nominal classifiers like 一桶 *jat1 tung2 one-bucket* 'a bucket of' to become 擔咗一桶水 *daam1 zo2 jat1 tung2 seoi2 carry-PERF-one-bucket-water* 'carried a bucket of water'. The verbal classifier modifies the action described by the predicate 擔 *daam1* 'to carry' while the nominal classifier modifies the nominal component 水 *seoi2* 'water'.

However, the Verb-Object Form ***縮水 suk1seoi2*** *shrink-water* 'to shrink through getting wet' cannot be modified by the same nominal classifier 一桶 *jat1 tung2* one-bucket 'a bucket of'. Since it is revealed by the syntactic tests that the nominal of the Verb-Object Form is not really the object to the V, this explains why the nominal 水 *seoi2* 'water' in the Verb-Object Form ***縮水 suk1seoi2*** 'to shrink through getting wet' is not quantifiable by the nominal classifier. Despite the fact that the nominal in this Verb-Object Form cannot be modified by a nominal classifier as indicated by the ungrammaticality of the expression ****縮***咗一桶***水 suk1*** *zo2 jat1 tung2* ***seoi2*** shrink-PERF-one-bucket-water, the Verb-Object Form still can be separated by a verbal classifier. This can be seen in the expression ***縮***咗一次***水 suk1*** *zo2 jat1 zi3* ***seoi2*** shrink-PERF-one-time-water 'shrunk through getting wet once' with the frequency phrase 一次 *jat1 zi3* one-time 'once' being interpreted to modify the V.

3.3 Conjoining

The unitary nature of the two separate elements of a Verb-Object Form is apparent in the phenomenon of conjoining. Works such as Bodomo (1998), Ackerman and Lesourd (1997) have used conjoining as a test for complex predicates which are also manifestations of two (or more) units functioning as a single unit. To join sentences in Cantonese, the word 同 *tung4* 'and' can be used. Firstly, let us take two verb phrases in (7) as an example:

[6] 3.SG - third person singular

(7) 爬山 爬石

paa4saan1 *paa4sek6*

climb-mountain climb-rock

'climb mountains' 'climb rocks'

We can join the two verb phrases 爬山 *paa4 saan1* *climb-mountain* 'climb mountains' and 爬石 paa4 sek6 *climb-rock* 'climb rocks' together by deleting the head verb 爬 *paa4* 'climb' of the second verb phrase and using the conjunction 同 *tung4* 'and' to link them up as in (8):

(8) 爬 山 同 石

paa4 *saan1* *tung4* *sek6*

climb mountain and rock

'climb mountains and rocks.'

The two verb phrases 爬山 *paa4 saan1*'climb mountains' and 爬石 *paa4 sek6* 'climb rocks' can be conjoined like the above because the nouns 山 *saan1* 'mountain' and 石 *sek6* 'rock' in these two phrases are subcategorized for by the common matrix verb 爬 *paa4* 'climb'.

However, it is impossible for us to conjoin the verb phrase 爬山 *paa4 saan1* 'climb mountains' and the Verb-Object Form ***爬頭 paa4tau4*** 'to overtake' shown in (9):

(9) 爬山 ***爬頭***

paa4 saan1 ***paa4 tau4***

climb-mountain climb-head

'climb mountains' 'to overtake'

If we join together the phrase and the Verb-Object Form in the same manner as we join the two verb phrases in the previous sentence, it will give us the following ill-formed expression in (10):

(10) *爬 山 同 頭

paa4 *saan1* *tung4* *tau4*

climb mountain and head

The Verb-Object Forms are identified by their non-compositional semantics. When the syntactic environments impose a compositional interpretation, the meaning of the Verb-Object Form as a complex predicate is not available. Conjoining (or coordination) in this case involves parallel constructions sharing a single grammatical relation to the remaining elements of the sentence. In (10), two conjoined NPs are governed by the same verb, one of the conjuncts has a literal reading 爬山 *paa4 saan1* 'climb mountain' Since a single token in a sentence cannot carry two different interpretations, the meaning of the predicate ***爬頭 paa4tau4*** *climb-head* 'to overtake' will not be available unless it is derived from the compositional meaning of the verb *爬 paa4 'climb'*. The expression in (10) illustrates that when ***爬頭 paa4tau4*** *climb-head* 'to overtake' takes a conjoined NP, the only possible reading is the literal *爬 paa4 'climb'* and this results in an awkward meaning of the verb phrase 'to climb somebody's head'. The ungrammaticality results from the deletion of the Verb Object Form's verbal which is apparently different from the verbal of the verb phrase in nature though they have the same morphological form. The deletion destroys the unitary nature of the Verb-Object Form ***爬頭 paa4tau4*** *climb-head* 'to overtake', deleting either one of the components makes it impossible to convey the meaning (non-compositional in nature) of the predicate. This shows that the nominal is not something that the verbal subcategorizes for.

3.4 Transitivity of the verbal

The transitivity (or intransitivity) of the verbal provides another evidence that supports one to subscribe to the view that the nominal is not the object of the verbal.

A verb can either be transitive or intransitive. In the case of VOC, even the verbal can be viewed as a transitive verb, the nominal still cannot be interpreted as the second, non-subject participant of the transitive verb. A transitive verb, according to the definition given by Li and Thompson (1981:157), is one whose meaning requires two participants and one of them is doing something to or directing some behavior at the other one. The participant who is doing something is the subject, and the one toward which or whom the behaviour is directed is the direct object. For instance, the verb 打 *daa2* 'hit' in 我打你 *ngo5 daa2 nei2* 'I hit you' is a two-place predicate, it requires two participants, one is the agent 我 *ngo5* 'I' and the other is the patient 你 *nei2* 'you'. Semantically, it is clear that the second participant, the direct object, of the verb would have to be some entities or things that you can perform the 'hit' action. However, in the VOC 打風 *daa4fung1* which means 'a typhoon is blowing' and which also has the same verbal 打 *daa2* 'hit' as in the utterance 我打你 *ngo5 daa2 nei2* 'I hit you', unless you interpret it with the literal meaning, there is no way to see the nominal 風 *fung1* 'wind' as the second, non-subject participant of the verb 打 *daa2* 'hit'. If you take it literally as {hit wind}, the meaning of 'a typhoon is blowing' can never be obtained.

Furthermore, in many cases, the verbal element of a VOC is not even transitive, that is, the meaning does not require two participants, like the two VOCs **溜冰 *lau6bing1*** 'to ski' and **瞓覺 *faan3gaau3*** 'to sleep', neither 溜 *lau6* 'to glide' nor 瞓 *faan3* 'to sleep' suggests that an object is required as they both are intransitive in nature.

Given that the nominal actually cannot be taken as the object and does not possess an argument status, the verbal and the nominal going together, rather than the verbal only, should be treated as a predicate as a whole.

3.5 Noun Incorporation (NI)

At first glance, NI seems to have many parallels to the VOC as their nominals are also like being 'absorbed' by the verbal and the resulting verb as a whole is intransitive and referring to a unitary concept. However, upon a closer inspection, the difference between them can be revealed. To illustrate this, let us first examine what NI refers to.

Typically, in the NI phenomenon of Cantonese (probably of Mandarin as well), there is a verb stem, plus a noun stem. The noun (before incorporation) is a syntactic argument of the verb (exemplified by (11a)). After incorporation, the compound (V+N), as a whole, refers to a unitary concept (exemplified by (11b)).

(11a) 佢 鍾意 飲 茶
keoi5 *zung1ji3* *jam2* *cha4*
3.SG like to drink tea
'S/he likes to drink tea.'

(11b) 佢 鍾意 飲茶
keoi5 *zung1ji3* *jam2cha4*
3.SG like to drinking tea and eating dimsum in a restaurant
'S/he likes to go to restaurant to drink tea and have dimsum.'

Before NI as in (11a), 飲 *jam2* 'drink' is the V stem and 茶 *cha4* 'tea' is the N stem which is the object argument of 飲 *jam2* 'drink'. Both stems are free morphemes. The compound obtained in (11b) is an intransitive verb referring to a highly institutionalized activity - drinking tea and eating dimsum in a restaurant. Thus, the fundamental difference between NI and VOC is that the noun in NI before incorporation is seen as a syntactic (object) argument of the verb and it is also a free morpheme which can stand alone on its own in a syntactic noun-class slot. On the contrary, the nominal in a VOC can barely be seen as an object argument as evidenced in the above sub-sections. It also cannot be counted as a free morpheme in the way the noun in NI does.

4 An LFG Analysis

The separable Verb-Object Forms present an 'analytic paradox' (Nash 1982. Cited in Ackerman & Lesourd 1997) for linguistic theory. On the one hand, the V and the N of a Verb-Object Form seem to constitute a single semantic word with a non-compositional (or specialized) meaning not summed up by the V and the N, suggesting that they have a status comparable to that of a normal lexical word; on the other hand, they permit extraneous materials to be inserted between V and N, making the Verb-Object Form appear to be a phrase. The question of how to represent a sentence which appears to have a syntactically complex structure with respect to certain phenomena and a syntactically simple structure regarding other phenomena poses a problem for theories of syntax. In this section, I shall demonstrate how Lexical-Functional Grammar (LFG) can tackle this paradox presented by the Cantonese data by its three parallel planes of syntactic representation.

4.1 The Architecture of Grammar

LFG was developed by Ronald Kaplan and Joan Bresnan (Bresnan 1982, Kaplan and Bresnan 1982) in the late 1970s/early 1980s as an alternative to Transformational Grammar in the search for a Universal Grammar. It differs radically from Transformational Grammar in the sense that LFG is non-derivational, non-transformational. The distinctiveness of LFG's architecture lies in its three parallel levels of representation which are the c(onstituent) structure, f(unctional) structure and a(rgument) structure. C-structure indicates the hierarchical composition of words into larger units or phrasal constituents, f-structure expresses the grammatical functions of a sentence and a-structure encodes the predicate-argument relations of a sentence. C-structure is linked to f-structure by a set of principles called Structure-Function Correspondence and f-structure is connected to a-structure by a theory called Lexical Mapping Theory. The diagram (Bodomo 2001) below in (12) visually shows the relationship between the three levels:

(12)

C-structure

| Structure-Functional Correspondence

F-structure

| Lexical Mapping Theory

A-structure

(Bodomo 2001)

As noted from the above diagram, the f-structure, due to its ability of accounting for cross-linguistic variations, can be seen as the point of intersection to bridge over the c-structure and the a-structure.

Within Lexical-Functional Grammar, which encodes distinct levels for representing the grammatical information of a sentence and factors the grammatical information of an expression into role, category and function, it is possible to reflect that the Verb-Object Forms are 'simple' in f(unctional)-structure encoding grammatical functions and 'complex' in c(onstituent)-structure encoding precedence and dominance relations.

To concretize the analysis of Cantonese VOCs in this formal syntactic framework, I shall take the following sentence in (13) for illustration.

(13)	我	游咗	三	個	鐘頭	水	(游水 ***jau4seoi2*** ‘swim’)
	ngo5	***jau4**-zo2*	*saam1*	*go3*	*zung1tau4*	***seoi2***	
	1.SG	swim-PERF	three	CL[7]	hour	water	
	‘I swam for three hours.’						

4.2 Argument Structure

Argument structures encode lexical information about the number of arguments, their syntactic type, and their hierarchical organization necessary for the mapping to syntactic structure. I have already demonstrated the non-argument status of the N and that there is no subcategorization involved between the V and the N which are indeed functioning as a unit, namely a predicate, so the a-structure of sentence (13) is rather simple and straight forward as shown in (14).

(14) ***jau4seoi2*** ‘SWIM’ < agent >

The above argument structure dictates that the sentence in (13) has a predicate 游水 *jau4seoi2* ‘swim’ which requires only one participant and it is the agent 我 *ngo5* ‘I’.

4.3 Constituent Structure

4.3.1 Annotated Phrase Structure Rules

Constituent structures encode linear order, hierarchical groupings, and syntactic categories of constituents. The first step to construct a c-structure for the example sentence is to provide annotated phrase structure rules (PSRs) as inputs. These rules consist of context-free skeletons annotated with schemata of functional equations which specify grammatical functions and direct the application of unification. The annotated PSRs I suggested for (13) are stated in (15):

(15)					
	a.	S →	DP $(\uparrow \mathrm{SUBJ})=\downarrow$	VP $\uparrow=\downarrow$	
	b.	VP →	V $\uparrow=\downarrow$	DP $(\uparrow \mathrm{ADJ})=\downarrow$	N $\uparrow=\downarrow$
	c.	DP →	D $\uparrow=\downarrow$	CLP $\uparrow=\downarrow$	
	d.	CLP →	CL $\uparrow=\downarrow$	NP $\uparrow=\downarrow$	
	e.	NP →	N $\uparrow=\downarrow$		

In LFG, an up-arrow ‘$\uparrow$’ refers to the grammatical function represented by the mother node and a down-arrow ‘$\downarrow$’ to the function represented by the current node. In (15a), the equation under the NP node stipulates that the node stands for the SUBJ function of its mother which is ‘S’ (the entire sentence). The equation under VP stipulates that it bears information about the same grammatical function as its mother. Likewise, in (15b), the equation under VP also stipulates that it bears the same grammatical function as its mother, S. However, it looks peculiar that the N node in (15b) which is not the lexical head in the VP also receives the equation $\uparrow=\downarrow$. There is justification for this annotation. In general, the LFG annotation of functional equations is dictated by how to properly represent grammatical information on each node, that is, by determining what partial information one constituent node contributes to f-structures. In LFG, functional heads are annotated with the equation $\uparrow=\downarrow$ (Zaenen 1983). So, the equation under the N node in (15b) would suggest that the N is also a functional head. This equation not only explicitly states that whatever

[7] CL - classifier

grammatical information specified under the N node is passed up to its mother node, but also requires that grammatical information represented on this node be unified with the grammatical information of its mother. Thus, when $\uparrow=\downarrow$ marks the N as a functional head of the construction, it formally stipulates that the grammatical information represented by this c-structure node belongs to the same f-structure represented by its mother. In terms of grammatical functions, annotating $\uparrow=\downarrow$ to more than one sister node requires that the grammatical information encoded on each be unified to form one single function at f-structure. In light of this, it correctly follows that the N shares the same f-structure with the VP as it is part of a discontinuous predicate of the construction. The lexical heads V, D, CL, and N in (15b), (15c), (15d) and (15e) respectively are all annotated with the same equation $\uparrow=\downarrow$ which means that they pass their lexical information (provided from the lexicon) up the tree to their mother nodes. The equation ($\uparrow$ADJ)=$\downarrow$ under the DP node in (15b) stipulates that the node stands for the ADJ(UNCT) function of its mother. The equation $\uparrow=\downarrow$ on the CLP node in (15c) and the NP node in (15d) means that they are co-heads to the D node and to the CL node respectively and share the same f-structure as their mothers.

4.3.2 Lexical Entries

In addition to information about syntactic functions, the grammatical representation for the example sentence also includes information about such grammatical features as tense, aspect and number. This kind of information comes from the schemata in the lexicon. The lexical entries in (16) are those that I postulate for the example sentence.

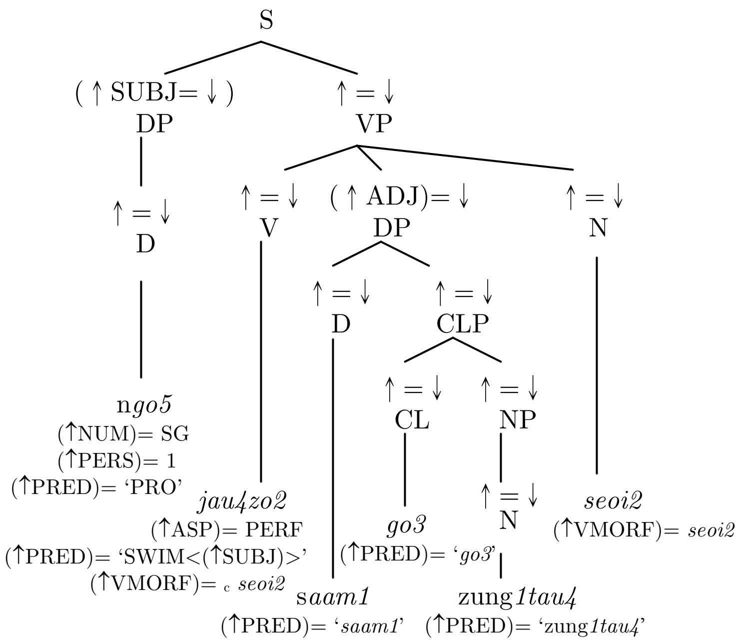
(16)	*jau4zo2*	V	($\uparrow$ASP)= PERF ($\uparrow$PRED)= ‘SWIM<($\uparrow$SUBJ)>’ ($\uparrow$VMORF)= $_c$ *seoi2*
	seoi2	N	($\uparrow$VMORF)= *seoi2*
	ngo5	D	($\uparrow$NUM)= SG ($\uparrow$PERS)= 1 ($\uparrow$PRED)= ‘PRO’
	saam1	D	($\uparrow$PRED)= ‘*saam1*’
	go3	CL	($\uparrow$PRED)= ‘*go3*’
	zung1tau4	N	($\uparrow$PRED)= ‘*zung1tau4*’

The highlight of the above lexicon is on the first two entries. The crucial characteristic of VOCs is their non-literal and non-compositionality of meaning (neither the verbal nor the nominal alone can fully represent the predicate) despite their noncontiguous position. This property would have to be captured by lexical entries. It is only the co-occurrence of the V and the N that yields the meaning of ‘swim’ for the Cantonese predicate 游水 *jau4seoi2*. As shown in (16), the binding force between the verbal 游 *jau4* and the nominal 水 *seoi2* comes from the constraining equation ($\uparrow$VMORF)= $_c$ *seoi2*. A constraining equation, marked by subscripting the letter *c* to the equal sign like ‘= $_c$’, does not create new attribute-value pair but constrain information coming from somewhere else. The attribute specified on the left-hand side of the equation is required to have the exact value specified on the right-hand side of the equation. Otherwise, the f-structure would be rendered ungrammatical. In other words, the constraining equation would invoke checking on that attribute at the final f-structure. So, the constraining equation ($\uparrow$VMORF)= $_c$ *seoi2* requires that the VMORF attribute be present with the value ‘*seoi2*’, the VMORF value uniquely stipulated on the nominal 水 *seoi2*. This ensures that the reading of ‘swim’ is only available with the co-occurrence of this designated nominal 水 *seoi2*. The constraining equation and hence the f-structure would be violated either if the value is not specified at all or if it is specified differently from what the constraining equation requires. The two pieces of lexically encoded grammatical information coming from the constraining equation and the functional equation ($\uparrow$VMORF)=*seoi2* under the lexical entry of 水 *seoi2* represent the fact that the full meaning of the predicate in the example sentence (13) depends on the unification of information carried by the two components.

4.3.3 An Illustration

With annotated rules in (15) and lexical entries in (16), an annotated c-structure representation for sentence (13) can be drawn up in (17) below.

(17)



'I swam for three hours.'

4.4 Functional Structure

The f-structure is the most expressive level of grammatical representation in LFG. LFG explicitly represents grammatical relations and refers to them as grammatical functions. This is done with the functional equations annotated to the c-structure which map a c-structure to a functional structure, where the grammatical relations of the string are formally represented. The f-structure for the example sentence is now demonstrated as follows in (18).

(18)

$$\left[\begin{array}{ll} \text{SUBJ} & \left[\begin{array}{ll} \text{PERS} & 1 \\ \text{NUM} & \text{SG} \\ \text{PRED} & \text{‘PRO’} \end{array}\right] \\ \text{PRED} & \text{PRED ‘SWIM<(}\uparrow\text{SUBJ)>’} \\ \text{ASP} & \text{PERF} \\ \text{ADJ} & [\textit{saam1 go3 zung1tau4}] \\ \text{VMORF} & \textit{seoi2} \end{array}\right]$$

The f-structure (18) specifies that the predicate-argument structure of the sentence contains one argument, i.e. the SUBJ function. The content of this argument is also specified in the f-structure by the inner f-structure assigned as the value to the attribute SUBJ. The ADJ(UNCT) function, carrying the semantic content of '*saam1 go3 zung1tau4*' which means 'three hours', is specified as an adjunct of the sentence, adding further information to the entire predicate about the duration of the action.

Conclusion

The 'Verb-Object Compound' is well-known for its contradictory behaviour with regard to the two major linguistic domains of syntax and morphology. The verbal and the nominal in combination making up the compound are semantically unitary, like a normal lexical item; however, they allow insertion of extraneous materials such as expressions of frequency/duration.

My goal in this paper has been to elucidate the non-object and non-argument status of the nominal of the Verb-Object Forms and to show that how the theoretical framework of Lexical-Functional Grammar handles lexical discontinuity presented by Cantonese VOCs. On the one hand, LFG is able to show the non-concatenating position of the two items in the c-structure; on the other, it is able to capture the unitary status of the VOC predicate in the f-structure through the mediation of a constraining equation.

References

Ackerman, F. & Lesourd, P., 1997. Toward a Lexical Representation of Phrasal Predicates. In Alsina, A., Bresnan, J. & Sells, P. *Complex Predicates*, 67-106. Stanford, Calif.: CSLI Publications.

Bodomo, A. B., 1998. Serial verbs as complex predicates in Dagaare and Akan. In Maddieson and Hinnebusch (eds.) *Language History and Linguistics Description in Africa: Trends in African Linguistics,* 2:195-204. Trenton, N.J.: Africa World Press.

Bodomo, A. B., 2001. Lecture Notes on Theories of Grammar. ms. University of Hong Kong.

Bresnan, J., 1982. *The Mental Representation of Grammatical Relations*. Cambridge, MA: MIT Press.

Bresnan, J., 2001. *Lexical-Functional Syntax*. Massachusetts: Blackwell Publishers Inc.

Chao, Y.-R., 1968. *A Grammar of Spoken Chinese.* California: University of California Press.

Huang, J.C. -T., 1983. Phrase structure, lexical ambiguity, and Chinese compounds. *Journal of the Chinese Language Teachers Association* 19(2):53-78.

Kaplan, R. & Bresnan, J., 1982. Lexical Functional Grammar: a Formal System of Representation. In Bresnan, J., ed., *The Mental Representation of Grammatical Relations*, 173-281. Cambridge, Mass.: MIT Press.

Li, N. C. & Thompson, A. S., 1981. *Mandarin Chinese: A Functional Reference Grammar.* California: University of California Press.

Packard, J. L., 2000. *The Morphology of Chinese: a linguistic and cognitive approach.* Cambridge: Cambridge University Press.

Zaenen, A., 1983. On syntactic binding. *Linguistic Inquiry*, 14:469-504.